



**Intent, Implementation and Impact
Long Term Plans & Progression
EYFS, KS1 and KS2 – September 2025**



This policy has been developed and ratified through the lens of a school vision: Believe, Achieve, Inspire'.

At Upper Nidderdale Primary Federation, we will all approach everything we do in the CHAMPS way, helping every child flourish into a caring, confident and resilient young person who has a **love of learning** and upholds our CHAMPS Values:

Community - Treat others as we would want to be treated ourselves

Hope – Giving confidence in what we can contribute and achieve together

Aspiration - Believe that we can be the best version of ourselves in all that we do

Mission – Living with purpose and commitment to making a positive difference

Perseverance – Not everything comes easily – keep trying to reach your goals and dreams

Shine – Let your light shine on yourself and others

As Rights Respecting schools, our intents are based around the following articles;

Article 23

You have the right to special education if you have a disability.

Article 28

All children have the right to a good quality education.

Article 29

All children have the right to an education that helps to develop their talents and abilities.

Intent

Intent

At The Upper Nidderdale Primary Federation believe that Mathematics is a key life skill that enables an individual to participate fully as a member of society. Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of Mathematics and a sense of enjoyment and curiosity about the subject.

The aims and intent of our Mathematics curriculum are to support pupils to:

- **Have rich and enjoyable experiences** by providing the knowledge, skills and understanding that enable all our pupils to flourish in society and be fully prepared for the next stage in their learning.
- **Develop positive and confident attitudes towards mathematics.**
- **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof.
- **Solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.
- **Develop the correct mathematical vocabulary.**
- **Work independently and collaboratively.**
- **Use technology to develop mathematical concepts.**
- **Use and apply their mathematical knowledge to real-life contexts.**

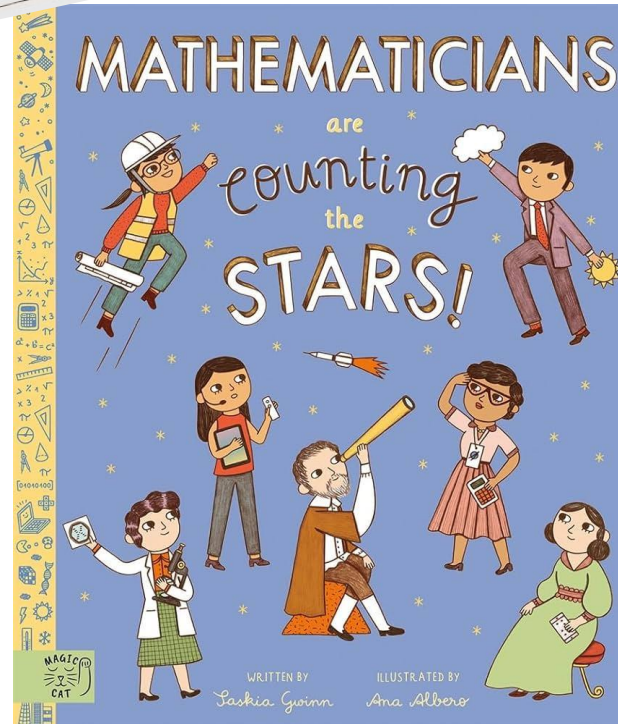
Intent

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

The Upper Nidderdale Primary Federation Mathematics Curriculum is based on the 2014 expectations and aims of the National Curriculum, and it incorporates the 'Ready to Progress' criteria set out in the DfE non-statutory guidance (2020) and the 2021 statutory framework for the Early Years foundation Stage (EYFS) and the Early Years 'Development Matters' guidance (2020). This framework provides a structure for the teaching of Mathematics. It sets out the minimum curriculum expectations of each year group, showing how content should be carefully sequenced year on year via our progression map.

Provision for our pupils who are working at a greater depth within their year group standard is paramount to improving and maintaining high standards, and consequently guidance regarding the development of greater depth mathematicians is also integral to this intent. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content.

Intent



Intent

At The Upper Nidderdale Primary Federation, all classes have a copy of the book, 'Mathematicians as counting the Stars.' We use this book because it exposes children the many areas of life that are affected by Maths, as well as presenting them with an array of careers that can be pursued through the study of Mathematics.

We are committed to developing children's curiosity about Maths, as well as an appreciation of the beauty and power of Mathematics. We are dedicated to ensuring that children are able to recognise the importance of Maths in the wider world and that they are also able to use their mathematical skills and knowledge confidently, in a range of different contexts.

Our Maths curriculum aims to be a gateway to opportunity and to a fulfilling and prosperous life in modern Britain and beyond.

Planning Maths in a linear and linked way using the scaffold of Five Phases

Declarative knowledge		Procedural knowledge	Conditional Knowledge		
Understand	Facts	Skills	Explore	Apply	
The teachers show what the concept is and what it isn't. The ideas are introduced.	The teachers teach and the children practise facts Children will be expected to reason about the facts and attempt more complex problems in every lesson i.e. 'working backwards'	The teachers teach and the children practise methods, skills and strategies Children will be expected to understand the methods, strategies/skills conceptually and be able to explain how they work and their use.	The children use the content from the Unit of work to learn how to Reason i.e. pursue a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language	The teachers teach the context(s) and 'genre' for the application of skills, strategies and facts i.e. if money and length is being used as part of an addition/subtraction unit of work...	The children can apply with little adult help
Each phase will have its own relevant vocabulary & language structures that are to be taught & used					

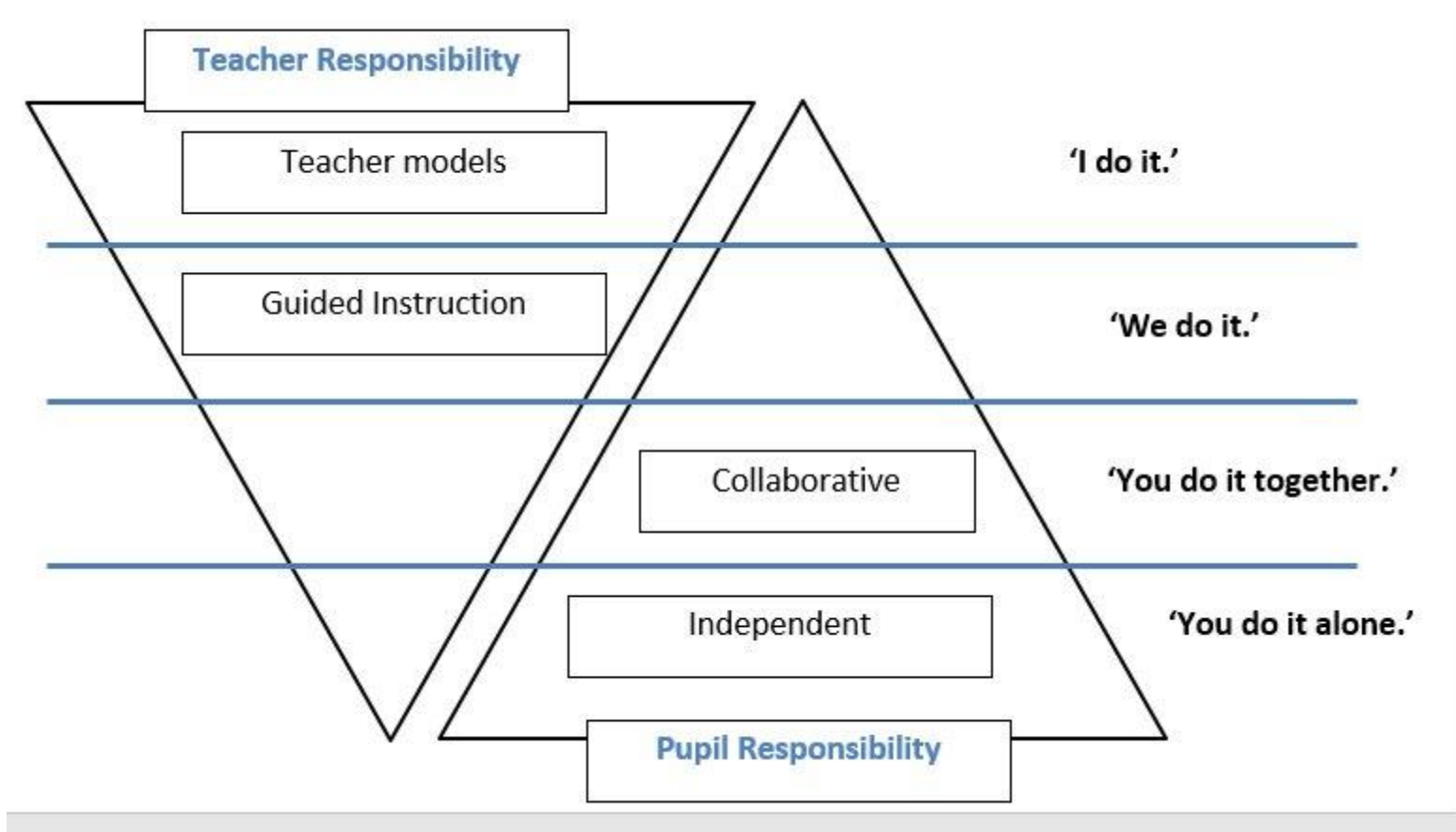
Implementation

Maths – Long Term Planning Rationale and Implementation

At The Upper Nidderdale Primary Federation, long-term planning in Maths is deliberately spaced and interleaved for revision and over-learning of the content. School sequencing materials are used as the starting point for curriculum planning, which is structured into four cycles. In each cycle, all strands of maths are taught.

- We believe mastery is achieved over time and through practice; this is reflected in the design and implementation of our maths curriculum and maths lessons.
- Retrieval Practice allows children to become secure within their knowledge and skills. Lessons and the work set by teachers are used to revise previous content. These are useful assessment opportunities: feedback is given to groups or the whole class as identified. Retrieval Practice starters are used to revise previous content and address misconceptions as identified through observing children's work and responses. Teachers have the freedom to determine the most useful learning to retrieve, and this is balanced alongside the Ebbinghaus forgetting curve of sequenced retrieval.
- Every day, children count and practise essential number knowledge. This is sequenced in such a way as to equip children with the most useful knowledge for subsequent reasoning lessons. The Number Knowledge session is systematic and deliberate: teachers will model, children practise through call and response
- Arithmetic is taught discretely from Year 3. This is sequenced to equip children with the calculation methods they need to apply in subsequent reasoning lessons.
- **In Reception, children build a secure foundation of mathematics. Learning is contextualised and given purpose within the continuous provision. Adults lead learning through direct teaching, group work and observations of independent learning. Subitisation, number sense and counting are a significant focus of the curriculum.**
- From Year 1, the Learning Journey model is used to sequence the reasoning learning of a given objective. Component steps on the learning journey are progressive. Within each step, children have the opportunity to acquire/refine, practise/apply, and extend/deepen their learning. Each level of challenge builds on prior learning and extends thinking. Problem solving and reasoning are inherent at every level of learning, and concrete manipulatives are used by all children to scaffold and deepen thinking.
- Component steps are intentionally planned so learning is cumulative and revisited across four-week cycles, to give all children the opportunity for deliberate practice and the tools to reach a greater depth standard when appropriate.

-
Children are guided through each part of a Maths lesson with a I do, we do, you to it approach.



How a lesson should look

- Activity on the board as they start the lesson. Recall or arithmetic focus.
- Monics/Mental Oral starter
- Recap of previous day
- Fluency and problem solving to be taught as separate lessons
- Follow I do, we do, you do approach for every question within a lesson

Multiplication Facts

- In Early Years and Year 1 children have the opportunity to count in steps throughout the half term following the long term plan.
- From Year 2 onwards the start of every Maths lesson is used to teach multiplication facts following the long term plan for each year group.
- High quality resources from NCETM Spine materials <https://www.ncetm.org.uk/teaching-for-mastery/mastery-materials/primary-mastery-professional-development/multiplication-and-division/> Top marks and Times table rockstars are used to support the teaching of this.
- Within a week's sessions the children are introduced to facts, given the opportunity to practise and then apply.
- Children are recognised and awarded weekly through stats on TTRS.

Maths – Implementation

Teaching and Learning –Assessment and Feedback

- **Starting points are identified through accurate teacher assessment and prior learning.**
- **Summative assessment is made towards the end of each cycle.**
- **Assessment checks what has been learnt (remembered) and analysis informs subsequent retrieval practice (teachers identify the things children need more opportunities to remember) and informs periods of consolidation. The curriculum model allows flexibility in order to respond to the strengths and needs of children.**
- **The intended learning is always the focus of actions in the classroom. Activities and resources are carefully chosen and deliberately designed to focus effort towards practising the learning intentions. Activities are rich in problem solving, reasoning and purposeful thinking. Children record their work with precision, care and pride.**
- **The working wall displays the steps of learning, and useful scaffolds and models for children to refer to in order to build independence.**
- **Concrete resources are the right of all children, not the crutch of the lowest attaining.**
- **Feedback is given in response to timely and continuous formative assessment in every lesson. Teachers use a range of formative assessment tools, including questions and observations to gauge children's level of understanding and knowledge. This is used to either offer support and scaffolds, or to give opportunities to deepen learning. Feedback is given in line with our feedback policy, including pink and purple marking, with green pen for the children to respond to the feedback given.**

Regular Practice/Recall of Core Skills

Core declarative and procedural knowledge will be practiced regularly and cyclically, regardless of what unit is currently being taught. This regular practice is a fundamental element of our Mathematics curriculum offer.

This approach supports pupils to know more and remember more, by providing regular opportunities to recall and revisit core knowledge. For example, even though a Year 2 class may be undertaking a spring term unit that focusses on Properties of Shape, pupils should still be recalling and practising their 2-, 5- and 10-times tables, as well as the core Year 2 procedural knowledge of column addition and subtraction.

Regular declarative and procedural practice can occur in a number of ways, including;

- Additional discrete arithmetic sessions.
- Additional discrete times tables/core number skills sessions
- DIRT time
- At the start of lessons as a warm-up and throughout the school day.

These sessions will never be 'holding activities,' They will give pupils the opportunity for deliberate practice and the overlearning of core mathematical declarative knowledge. Planned sessions will take place every Friday.

Recall of Core Skills

Core Mathematical Declarative Knowledge - EYFS and KS1

Reception	Year One	Year Two
• Represent and use number bonds and related subtraction facts within 10. • Add and subtract two single digit numbers up to 20 and count back to find the answer.	• Count, read and write numbers up to 100 in numerals. • Count in multiples of 2s, 5s and 10s. • Given a number, identify one more and one less. • Represent and use number bonds and related subtraction facts within 20. • Add and subtract one-digit and two-digit numbers to 10, including 0.	• Count in steps of 2, 3 and 5 and in 10s from any number, backwards and forwards. • Recognise the place value of each digit in a two-digit number. • Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100. • Add and subtract numbers using concrete objects, pictorial representations and mentally, including a two-digit number and 1s; a two-digit number and 10s; two-digit numbers and adding 3 one-digit numbers. • Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

Core Mathematical Declarative Knowledge - KS2

Year Three	Year Four	Year Five	Year Six
- Count from 0 in multiples of 4, 8, 50 and 100. - Find 10 or 100 more or less than a given number - Recognise the place value of each digit in a 3-digit number. - Add and subtract numbers mentally, including: a three-digit number and 1s; a three-digit number and 10s; a three-digit number and 100s. - Add and subtract numbers with up to 3 digits, using formal, written methods of column addition and subtraction. - Recall and use multiplication and division facts for the 2, 4, 5, 8 and 10 multiplication tables. - Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10) - Write and calculate mathematical statements for multiplication and divisions using the multiplication tables they know, including for two-digit times one-digit numbers, using mental and progressing to formal written methods.	- Count in multiples of 6, 7, 9, 25 and 1,000. - Find 1,000 more or less than a given number. - Add and subtract numbers with up to 4 digits using the formal written methods of column addition and subtraction where appropriate. - Recall multiplication and division facts for multiplication tables up to 12 x 12. - Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100). - Multiply two-digit and three-digit numbers by one-digit using formal written layout. - Divide numbers up to 3 digits by a one-digit number using the formal written method of short division without remainders.	- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000. - Add and subtract numbers mentally with increasingly large numbers. - Multiply numbers up to 4 digits by one or two-digit numbers using a formal written method, including long multiplication for two-digit numbers, - Multiply and divide numbers by mentally drawing upon known facts. - Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. - Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000. - Recall multiplication and division facts for multiplication tables up to 12 x 12.	- Add and subtract whole numbers with more than 4 digits including using formal written methods (column addition and subtraction) - Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. - Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division. - Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. - Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000. - Recall multiplication and division facts for multiplication tables up to 12 x 12.

Impact

Types of Mathematics Knowledge

Mathematical Knowledge

In the Department of Education **Research Review Series-Mathematics**, mathematical content was classified into three different types of knowledge for the purposes of explaining how pupils' mathematics schema should be developed over time. These three types of knowledge are;

- **Declarative Knowledge** is static in nature and consists of facts, formulae, concepts, principles and rules. All content in this category can be prefaced by the sentence stem '**I know that**' - e.g. I know that $8 \times 5 = 40$. I know that squares have four right angles.
- **Procedural Knowledge** is recalled as a sequence of steps. This includes methods and procedures such as how to perform long division or add fractions with the same denominator. All content in this category can be prefaced by the sentence stem '**I know how**' – e.g. I know how to do column addition. I know how to calculate the perimeter of a square,
- **Conditional Knowledge** gives pupils the ability to reason and solve problems. Useful combinations of declarative and procedural knowledge are transformed into strategies when pupils learn to match the problem types that they can be used for. All content in this category can be prefaced by the sentence stem '**I know when**' – e.g. I know when to use a bar model to solve a one-step money problem.

Mathematical Knowledge

Over time, as pupils learn and use declarative, procedural and conditional knowledge, the knowledge of relationships between concepts develops and their mathematics schema deepens. All Maths lessons at The Upper Nidderdale Primary Federation are driven by success criteria that are categorised by these three types of knowledge. These success criteria should not be overly specific, so they can apply to the whole lesson where possible.

For example;

Problem solving – multiplication

Discover



1 a) How many people are going to Bolton Towers?
b) How many people are going on a trip today?

Share

a) There are 56 people on each coach going to Bolton Towers.
There are 3 coaches.

56	56	56
----	----	----

$$\begin{array}{r} 56 \\ \times 3 \\ \hline 168 \end{array}$$

There are 168 people going to Bolton Towers.

b) We know that 168 people are going to Bolton Towers.

168	60	60	60	60
-----	----	----	----	----

Bolton Towers Adventure Park

To work out the total I did
 $56 + 56 + 56 + 60 + 60 + 60 + 60$.

There are 4 coaches going to Adventure Park.
There are 60 people on each coach.
 $60 \times 4 = 240$
There are 240 people going to Adventure Park.
 $168 + 240 = 408$
There are 408 people going on a trip today.

I used the answer from earlier and then used 6×4 to help me to do 60×4 .
Then I added my answers together.

The success criteria for this type of problem would be;

- I know my times tables facts (declarative knowledge)
- I know how to do short multiplications (procedural knowledge)
- I know how to do column addition (procedural knowledge)
- I know when to use a bar model to solve two-step word problems (conditional knowledge)

SEND adoptions
in Maths

Special Educational Needs and Disabilities (SEND)

Taken from the 2020 DfE document, Teaching Mathematics in Primary Schools."

Pupils should have access to a broad and balanced curriculum. The National Curriculum Inclusion Statement stated that teachers should set high expectations for every pupil, whatever their prior attainment. Teachers should use appropriate assessment to set targets which are deliberately ambitious. Potential areas of difficulty should be identified and addressed at the outset. Lessons should be planned to address potential areas of difficulty and to remove barriers to pupil achievement. In many cases, such planning will mean that pupils with SEN and disabilities will be able to study the full national curriculum. The guidance in this document will support planning for all SEND pupils by highlighting the most important concepts within the national curriculum so that teaching and targeted support can be weighted towards these.

Curriculum Sequencing of Knowledge

Mathematical Sequencing of Knowledge

Sequencing the learning of knowledge within a unit of learning requires careful consideration. The required facts – **declarative knowledge** (e.g. times tables) need to be learnt and easily recalled, before using these when learning and mastering the **procedural knowledge** (e.g. short multiplication).

Once pupils are proficient in the **declarative and procedural knowledge**, pupils should then move onto tackling problem solving and reasoning strategies – **the conditional knowledge**. In short, proficiency in conditional knowledge requires proficiency in associated **procedural knowledge**, which in itself, requires fluency in the underlying **declarative knowledge**. Problem solving (**conditional knowledge**) requires pupils to hold a line of thought. It is not easy to learn, rehearse of experience if the facts and methods that form part of the strategy for solving a problem type are unfamiliar and take up too much working memory.

We ensure the regular, systematic rehearsal of core **declarative knowledge and procedural knowledge** by routinely revisiting this knowledge – e.g. times tables practice and discrete age-related arithmetic session to allow pupils to experience success when problem solving and reasoning. Pupils need procedural fluency to enable them to become proficient problem solvers.

Five Phased Approach to Maths

Start the planning by deciding on what you want the children to do independently by the end of the unit or by looking for gaps they have in their facts and strategies to decide on the content for the declarative and procedural knowledge. Or base this on your school's Long Term Plan for Maths. Also include your analysis of the children's needs which could be from a recent test or through observations or a mixture of the two. Then unpick all the facts, vocabulary, skills and strategies that the children will need in order for them to access problems given to them in the independent stage or think about how they will apply the facts/strategies they need to learn.

The understanding phase of the Unit can include historical contexts of the ideas being looked at as well as ensuring that misconceptions are considered. This can be a short stage of the unit of work or longer if needed.

The facts, skills, strategies and vocabulary are actively taught at the beginning of the unit and these are usually drawn from the NUMBER sections of the NC (although there are elements of skills and facts that need to be learned from the MEASUREMENT and GEOMETRY domains also such as how to measure with a ruler and what the names and properties of shapes are etc.)

Declarative Knowledge		Procedural Knowledge	Conditional Knowledge	
PHASE 1: Understand	PHASE 2: Facts	PHASE 3: Skills	PHASE 4: Explore	PHASE 5: Apply
The teachers show what the concept is and what it isn't. The ideas are introduced.	The teachers teach and the children practice the facts. Children will be expected to reason about the facts and attempt more complex problems in every lesson i.e. 'Working backwards'	The teachers teach and the children practice methods, skills and strategies. Children will be expected to understand the methods strategies and skills conceptually and to be able to explain how they work and their use.	The children use the content from phase 1 & 2 in puzzles and investigations as well as being given time to work on their explanation skills. (specifically deep learning and reasoning is promoted)	The teachers teach the context and genre for the application of skills, strategies and facts i.e. If money and length is being used as part of addition and subtraction The children can then apply with little adult help
Each phase will have its own relevant vocabulary and language structures that are to be taught & used.				

Monics

To support the building of proficiency and automaticity in number facts, all children from Year 5 and below participate in daily Monics sessions. This is the first 15 minutes of the maths session. Monics, is 'Maths Phonics', and is building the foundations of maths for all pupils, so that they can 'read' maths.

The progression is found below.

Progression:	EYFS (no number sentences)	Y1 (inc. number sentences)	Y2	Y3	Y4/5/6
Subsiting images of amounts (both perceptual and conceptual), using the arrangements of images to prove the properties and proximity of amounts.	Dice patterns up to six.	Dice, domino, odd/even patterns up to ten (also triangle/square arrangements).	Odd and even using plates images as well as tens frames.	Odd and even proofs using plate and ten frame patterns.	Images up to five where all plate patterns are unshaded.
	Odd and even up to six				Use of above images where each circle represents a multiple i.e. 'Each circle represents four. How much is the 'plate' image worth?'
	One more and one less up to six.	One more and one less up to ten.	Five plus an amount (six to ten) using plates, dominoes and tens frames.	Five plus an amount up to ten then ten plus an amount up to twenty.	Images of six to ten where all plate patterns are unshaded. Focus on what the arrangement proves about the numbers.
	Doubles up to double five.	Doubles up to double five.	One more/less up to ten.	Doubles and halves (up to ten and multiples of ten up to one hundred).	Use of above images where each circle represents a multiple i.e. 'Each circle represents four. How much is the 'plate' image worth?'
	Domino and odd/even patterns for seven, eight, nine and ten.	Odd and even up to ten.			Eleven to twenty using tens frames.
	All odd and even numbers up to ten.	Five plus an amount (six to ten).	Base ten images for multiples of ten.	Multiples of ten that compliment to 100 using images of a hundred square.	Doubles and 'five plus an amount' used where circles (shaded and unshaded) represent different amounts. Use of the hundred square images to represent fractions, decimals and percentages.
	Numbers from six to ten and how they relate to five.	Bonds of ten.	Base ten images for two-digit numbers.	Images of a hundred square to learn all other complements of 100 where there is value in the ones place.	
	Bonds of ten.	Eleven to twenty.	Base ten images for halves of multiples of ten.		

Connected Knowledge - Progression Map

Key Stage One &
Key Stage Two

Number & Place Value

Number and Place Value

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

*Green - Non-statutory 'Ready to Progress' criteria

Counting	Reception	Year One	Year Two	Year Three	Year Four	Year Five	Year Six
	- Children count reliably with numbers from one to 20 and place them in order. - Understand the 'one more/one less than' relationship between consecutive numbers. - Verbally count beyond 20, recognising the pattern of the counting system.	- Count within 100, forwards and backwards, starting with any number. - Count to and across 100, forwards and backwards; beginning with 0 or 1, or any given number. - Given a number 1-100, identify one more/less. - Count backwards and forwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count backwards through the odd numbers.	- Count in steps of 2, 3 and 5 from 0, and in tens from any given number, forward or backward. - Find 10 more or less than a given number.	- Count from 0 in multiples of 4, 8, 50 and 100. - Find 10 or 100 more or less than a given number.	- Count in multiples of 6, 7, 9, 25 and 1,000; count in steps of 10 and 100 from any given number beyond 1000. - Find 1,000 more than a given number.	Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000.	

Counting

Number and Place Value

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

*Green - Non-statutory 'Ready to Progress' criteria

Reading & Writing Numbers	Reception	Year One	Year Two	Year Three	Year Four	Year Five	Year Six
	Read and write numbers to 20 in numerals.	Read and write numbers to 100 in numerals; read and write numbers from 1 to 20 in numerals and words.	Read and write numbers to at least 100 in numerals and in words.	Read and write numbers up to 1000 in numerals and words.	- Read and write numbers to at least 10,000 in numerals and words. - Read Roman Numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.	- Read and write numbers to at least 10,000,000 in numerals and words. - Read Roman Numerals to 1000 (M) and recognise years written in Roman Numerals.	Read and write numbers up to 10,000,000 in numerals and words.

Reading & Writing Numbers	Reception	Year One	Year Two	Year Three	Year Four	Year Five	Year Six
	Read and write numbers to 20 in numerals.	Read and write numbers to 100 in numerals; read and write numbers from 1 to 20 in numerals and words.	Read and write numbers to at least 100 in numerals and in words.	Read and write numbers up to 1000 in numerals and words.	- Read and write numbers to at least 10,000 in numerals and words. - Read Roman Numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.	- Read and write numbers to at least 10,000,000 in numerals and words. - Read Roman Numerals to 1000 (M) and recognise years written in Roman Numerals.	Read and write numbers up to 10,000,000 in numerals and words.

Number and Place Value

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Identifying, Estimating and Representing Numbers	- Have a deep understanding of number up to 10, including the composition of each number. - Subitise (recognise quantities without counting) up to 5. - Identify, represent and estimate numbers 1-20.	- Identify and represent numbers up to 100 using objects and pictorial representations including the number line. - Reason about the location of numbers to 20 within the linear number system.	- Identify, represent and estimate number up to 100 using different representations including the number line. (e.g. place value cards, place value counters, dienes apparatus, number lines). - Reason about the location of any two-digit number in the linear number system, including identifying the previous and next multiple of 10.	- Identify, represent and estimate numbers up to 1000 using different representations (e.g. place value cards, place value counters, dienes apparatus, number lines). - Reason about the location any three-digit number in the linear number system, including identifying the previous and next multiple 100 and 10. - Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples with 2, 4, 5 and 10 equal parts.	- Identify, represent and estimate numbers up to 10,000 using different representations (e.g. place value cards; place value counters; dienes apparatus; number lines) - Reason about the location of any four-digit number in the linear number system, including identifying the previous and next multiple of 1,000 and 100. - Divide 1,00 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts.	Reason about the location of any number up to 2 decimal places in the linear number system, including identifying the previous and next multiple of 1 and 0.1.	- Reason about the location of any number p to 10 million, including decimal fractions in the linear number system and round numbers as appropriate, including in contexts. - Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts.

Identifying, Estimating and Representing Numbers

Number and Place Value

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Comparing Numbers	Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity.	Use the language of; equal to, more than, less than (fewer), most, least including comparing using < > and =	Compare and order numbers from 0 to 100; use <, > and = signs.	Compare and order numbers up to 1,000 (including using the < and > signs)	Order and compare numbers beyond 1,000 (including using < and > signs)	Order and compare numbers to at least 1,000,000 (including the < and > signs)	Order and compare numbers up to 10,000,000 (including the < and > signs).

Number and Place Value

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

Number and Place Value

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

Number and Place Value

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

Place Value	Reception	Year One	Year Two	Year Three	Year Four	Year Five	Year Six
		Begin to recognise the place value in numbers beyond 20 by reading, writing and comparing numbers, supported by concrete and pictorial representations	- Recognise the place value of each digit in a two-digit number (tens, ones) including partitioning number in different ways e.g. $23 = 20 + 3$ and $23 = 10 + 13$) and beginning to understand 0 as a place holder. - Compose and decompose two-digit numbers using standard and non-standard partitioning.	- Recognise the place value of each digit in a three-digit number (hundreds, tens and ones) and compose and decompose three-digit numbers using standard and non-standard partitioning e.g. $146 = 100 + 40 + 6$ and $146 = 130 + 16$). - Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three-digit multiples of 10.	- Recognise the place value of each digit in a four-digit number (e.g. say that the 4 in 3467 is worth 400). Compose and decompose four-digit numbers using standard and non-standard partitioning. - Know that 10 hundreds are equivalent to 1,000, and that 1,000 is 10 times the size of 100; apply this to identify and work out how many 100s there are in other four-digit multiples of 100.	- Determine the value of each digit in numbers up to 1,000,000. - Compose and decompose numbers up to 1,000,000 using standard and non-standard partitioning.	Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers up to 20 million using standard and non-standard partitioning.

Number and Place Value

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Rounding Numbers					- Reason about the location of any four-digit number in the linear number system, including identifying the previous and next multiple of 1,000, 100 and 10 and rounding to the nearest of each. - Round any number to the nearest 10, 100 or 1000	- Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000. - Reason about the location of any number with up to 2 decimal places in the linear number system, including identifying the previous and next multiple of 1 and 0.1 and rounding to the nearest of each.	- Round any whole number to a required degree of accuracy. - Reason about the location of any given numbers up to 10 million, including decimal fractions, in the linear number system, and round numbers as appropriate including in contexts.

Number and Place Value

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Negative Numbers		.			Count backwards through zero to include negative numbers.	Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero.	Us negative numbers in context and calculate intervals across zero.

Number and Place Value

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Problem Solving	Solve problems involving all of the previous statements.	Solve problems involving all of the previous statements.	Use place value and number facts to solve problems.	Use place value and number facts to solve problems involving all of the above and with increasingly large positive numbers.	Use place value and number facts to solve problems involving all of the previous statements.	Use place value and number facts to solve problems involving all the previous statements.	Use place value and number facts to solve problems involving all of the previous statements.

Addition,
Subtraction,
Multiplication
& Division

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Number Bonds	Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.	- Develop fluency in addition and subtraction facts within 10. - Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers. - Represent and memorise, use and reason with number bonds and related subtraction facts within 20 (e.g. $9 + 7 = 16$; $16 + 7 = 9$; $7 = 16 - 9$)	- Secure fluency in addition and subtraction facts within 10, through continued practice. - Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100 (e.g. using $3 + 7 = 10$, $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $30 + 70 = 100$, $100 - 70 = 30$ and $70 + 100 - 30$)	Secure fluency in addition and subtraction facts that bridge 10, through continued practice.			

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Addition and Subtraction - Mental Methods and Understanding	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
		- Add and subtract one-digit and two-digit numbers to 20, including zero. - Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs and relate additive expressions and equations to real-life contexts.	- Add and subtract across 10. - Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot. - Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more ..?" Add and subtract numbers mentally including; - A two-digit number and ones; - A two-digit number and tens; - Two two-digit numbers; - Adding three one-digit numbers.	- Calculate complements to 100. - Apply place-value knowledge to known additive and multiplication number facts (scaling facts by 10). Add and subtract numbers mentally including; - A three-digit number and ones. - A three-digit number and tens. - A three-digit number and hundreds. - Understand and use the commutative property of addition, and understand the related property for subtraction.	Apply place value knowledge to know additive number facts (scaling facts by 100)	- Apply place value knowledge to known additive number facts (scaling facts by 1 tenth or 1 hundredths). - Add and subtract numbers mentally with increasingly large numbers.	- Add and subtract numbers mentally with increasingly large numbers. - Use the compensation property of addition to complete equations such as, $25 + 35 = 27.5 + ?$ And help them to solve calculations such as $27.5 + 32.5$. - Use their knowledge of the order of operations to carry out calculations involving the four operations (e.g. BIDMAS)

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Addition & Subtraction - Written Calculations	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
	Using quantities and objects, add and subtract two single-digit numbers and count on or back to find that number.	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs and relate additive expressions and equations in real-life contexts (also in mental calculations)	Add and subtract numbers using concrete objects, pictorial representations, and mentally including; - A two-digit number and ones; - A two-digit number and tens; - Two two-digit numbers; - Adding three one-digit numbers - Add and subtract within a 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a two-digit number.	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.	Add and subtract numbers with up to 4-digits using the formal written methods of columnar addition and subtraction where appropriate.	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	- Understand that 2 numbers can be related additively or multiplicatively and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number. - Use a given or multiplicative calculation to derive or complete a related calculation using arithmetic properties, inverse relationships and place value understanding/

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Inverse, Estimating, Rounding Answers and Checking	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
			Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems (e.g. use addition to check subtractions)	- Understand the inverse relationship between addition and subtraction, and how both relate to the part-part-whole structure. - Estimate the answer to a calculation and use inverse operations to check answers.	Estimate and use inverse to check answers to a calculation.	Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy; round answers to a specified degree of accuracy.	Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements
 *Green - Non-statutory 'Ready to Progress' criteria

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements
*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Multiplication and Division Facts & Mental Methods		Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.	- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including odd and even numbers. - Show that multiplication of two number can be done in any order (commutative) and division of one number by another cannot, and use commutativity and inverse relations to develop multiplicative reasoning (e.g. $4 \times 5 = 20$ and $20 \div 5 = 4$). - Recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2, 5 and 10 multiplication tables.	- Continue to develop fluency in recalling multiplication facts, and corresponding division facts, 2, 5 and 10 multiplication tables. - Recognise products in these multiplication tables as multiples of the corresponding number. - Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. - Recognise products in these multiplication tables as multiples of the corresponding number.	- Recall multiplication and division facts for multiplication tables up to 12×12 and recognise products in multiplication tables as multiples of the corresponding number. - Combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations (e.g. $2 \times 6 \times 5 = 10 \times 6 = 60$) - Recognise and use factor pairs and commutativity in mental calculations. - Use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together three numbers. - Manipulate multiplication and division equations and understand and apply the commutative property of multiplication.	- Secure fluency in multiplication facts, and corresponding division facts, through continued practice. - Multiply and divide numbers by 10, 100 and 1000. - Multiply and divide numbers mentally, drawing upon known facts, applying all multiplication tables and related division facts frequently, commit them to memory and use confidently to make larger calculations. - Apply place value knowledge to know additive and multiplication number facts (scaling facts by 1 tenth or 1 hundredth)	- Continue to use all the multiplication tables to calculate mathematical statement in order to maintain fluency. - Use their knowledge of the order of operations to carry out calculations involving four operations (e.g. explore the order of operations using brackets)

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Properties of Numbers, Multiples, Factors, Squares, Cubes & Prime Numbers	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
						- Identify multiples and factors, including finding all facto pairs of a number (including common factors and common multiples, and express a given number as a product of 2 or 3 factors) - Know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers. - Establish whether a number up to 100 is prime and recall prime numbers up to 19. - Recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3).	Identify common factors, common multiples and prime numbers (including relating common factors to finding equivalent fractions)

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Multiplication and Division - Written Calculation			- Write and calculate mathematical statements multiplication and division within the multiplication tables and write them using the correct signs, working with a range of materials and contexts to relate to grouping and sharing discrete and continuous quantities to arrays and to repeated addition. - Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotative division)	Write and calculate mathematical statements for multiplication and division using multiplication tables that pupils know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods of short multiplication and short division.	- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. - Divide numbers up to 3 digits by a one-digit number using the formal written method of short division, without remainders. Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size.	- Multiply numbers up to 4 by one-digit or two-digit number using formal written method of short division and interpret remainders appropriately for the context. - Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.	- Multiply multi-digit numbers up to 4 digits by two-digit whole numbers using the formal written method of long multiplication. - Divide numbers up to 4 digits by a two-digit whole number using the formal written methods of short and long division as appropriate and interpret remainders as whole numbers remainders, fractions, decimals up to 2 decimal places or by rounding as appropriate for the context.

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements
 *Green - Non-statutory 'Ready to Progress' criteria

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements
 *Green - Non-statutory 'Ready to Progress' criteria

Addition, Subtraction, Multiplication & Division

*Black - National Curriculum Statutory Requirements
 *Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Problem Solving	Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations and missing number problems such as; 7 = ___ - 9 - Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher (including doubling, halving and quartering numbers and quantities)	Solve problems with addition and subtraction; - Using concrete objects and pictorial representations, including those involving numbers, quantities and measures. - Applying their increasing knowledge of mental and written methods. - Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	- Solve problems including missing number of problems, using number facts, place value and more complex addition and subtraction. - Solve problems including missing numbers problems involving multiplication and division, including integer scaling problems in which objects are connected to objects (e.g. 3 hats and 4 coats, how many different outfits?) - Apply known multiplication and division facts to solve contextual problems with different quotative and partition division.	- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. - Solve problems involving multiplying and adding, using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. - Solve division problems with two-digit dividends and one-digit divisors, that involve remainders, and interpret remainders appropriately according to the context.	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes. Solve problems involving multiplication and division, including scaling simple fractions and problems involving simple rates. Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. Use a given additive or multiplication to derive or complete a related calculation, using arithmetic problems, inverse relationships and place value understanding.	

Fractions (including Decimals & Percentages)

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Counting, Recognising, Finding & Writing Fractions	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
		- Recognise, find and name a half as one of two equal parts of an object, shape or quantity. - Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity. - Combine halves and quarters to make a whole.	- Count in fractions from 10, starting from any number and using $\frac{1}{2}$ and $\frac{2}{4}$ equivalence on the number line. - Recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity. - Write simple fractions for example $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$	- Interpret and write proper fractions to represent 1 or several parts of a whole that is divided into equal parts. - Reason about the location of any fraction within 1 in the linear number system. - Count up in tenths arise from dividing and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts (division) and in dividing one-digit numbers/quantities by 10; connect tenths to place value. - Recognise and write fractions of discrete set of objects; unit fractions and non-unit fractions with small denominators using known division facts (multiplication tables fluency)	- Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten. - Understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths. - Reason about the location of mixed numbers in the linear number systems.	Continue to practice counting forwards and backwards in simple fractions and backwards in simple fractions, including bridging) (e.g. on a number line) Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents. Continue to develop understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities (e.g. $\frac{4}{5}$ of 50)	

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Equivalence (including Fractions, Decimals & Percentages)	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
			Recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$	Recognise and show, using diagrams, equivalent fractions with small denominations.	- Recognise and show, using diagrams, families of common equivalent fractions; use factors and multiples to recognise equivalent fractions and simplify where appropriate (e.g. $\frac{6}{9} = \frac{2}{3}$) - Recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ - Recognise and write decimal equivalents of any number of tenths or hundredths. - Convert mixed numbers to improper fractions and vice versa.	- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths and understand that they have the same value and the same position in the linear number system. - Recall decimal fraction equivalents for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$. - Read and write decimal numbers as fractions (e.g. $0.71 = \frac{71}{100}$). - Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents. - Recognise the percent symbol (%) and understand that percent relates to 'number of parts per hundred'; write percentages as a fraction with denominator hundred and as a decimal.	- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination. - Associate a fraction with division and calculate decimal equivalents (for example 0.375) for a simple fraction e.g. $\frac{3}{8}$) - Recall and use equivalence between simple fractions, decimals and percentages including in different contexts.

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Compare & Order Fractions	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
				Compare and order unit fractions, and fractions with the same denominators.	Compare and order fractions with the same denominators including tenths and hundredths beyond one whole.	Compare and order fractions whose denominators are all multiples of the same number.	- Express fractions in common denomination and use this to compare and order fractions that are similar in value including fractions > 1 (e.g. order $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{13}{12}$) - Compare and order fractions with different denominators, including fractions greater than 1, using reasoning and choose between reasoning and common denominators as a comparison strategy.

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Adding & Subtracting Fractions	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
				Add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)	- Add and subtract fractions with the same denominator. - Add and subtract fractions with the same denominator beyond one whole (e.g. $\frac{3}{7} + \frac{5}{7} = \frac{8}{7}$) - Begin to add and subtract improper and mixed fractions with the same denominator, including bridging whole numbers.	- Add and subtract fractions with the same denominator and denominators that are multiples of the same number, including calculations that exceed 1 as a mixed number (e.g. $\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1 \frac{3}{6}$) - Recognise mixed numbers and improper fractions $\frac{1}{2}$ and convert from one form to another, write mathematical statements >1 as a mixed number (e.g. $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1 \frac{1}{5}$)	Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Multiplying & Dividing Fractions				Understand the relation between unit fractions as operators (fractions of), and division of integers.		Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams (e.g. $\frac{2}{6} \times 4 = \frac{8}{6} = 1 \frac{2}{6}$)	- Multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$). - Divide proper fractions by whole numbers (e.g. $\frac{1}{3} \div 2 = \frac{1}{6}$) - Use understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity.

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Decimal Rounding					Round decimals with one decimal place to the nearest whole number.	Round decimals with two decimal places to the nearest whole number and to one decimal place.	- Solve problems which require answers to be rounded to specified degrees of accuracy. - Round and estimate to predict and check the order of magnitude of answers to decimal calculations.

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Understanding, Reading, Writing, Comparing & Ordering Decimals					• Compare and order numbers with the same number of decimal places up to two decimal places. • Learn decimal notation and the language associated with it, including in the context of measurements, and relate decimal notation to division of whole numbers by 10 and 100.	• Read, write, order and compare numbers with up to three decimal places. • Recognise the place value of each digit in numbers with up to 2 decimal places and compose and decompose numbers with up to 2 decimal places using standard and non-standard partitioning. • Know that 10 tenths are equivalent to 1 one, 1 is 10 times the size of 0.1; 100 hundredths are equivalent to 1 one; 1 is 100 times the size of 0.01; 10 hundredths are equivalent to 1 tenth and 0.1 is ten times the size of 0.01.	• Identify the value of each digit in numbers given to three decimal places (e.g. 0.874 as 7 hundredths or 70 thousands).

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Adding & Subtracting Decimals						- Mentally add and subtract tenths and one-digit whole numbers and tenths. - Add and subtract decimals, including a mix of whole numbers and decimals (e.g. $3 - 2.58$), decimals with different numbers of decimal places (e.g. $0.247 + 5.7$), and complements of 1 (e.g. $0.83 + 0.17 = 1$)	- Mentally add and subtract hundredths and one-digit whole number and hundredths. - Increase fluency of adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different decimal places and compliments of 1.

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Multiplying & Dividing Decimals	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
					Find the effect of dividing a one or two-digit number by 10 and 100; identifying the value of digits in the answer as ones, tenths and hundredths.	Multiply and divide numbers by 10 and 100, giving answers up to three decimal places, identifying the value of the digits in the answer as ones, tenths and hundredths and thousandths.	- Multiply and divide numbers by 10, 100 and 1,000 giving answers up to three decimal places. - Multiply one-digit numbers with up to 2 decimal places. - Multiply and divide numbers with up to two decimal places by one- and two-digit whole numbers. - Use written division methods in cases where the answer has up to two decimal places.

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Fraction, Decimal and Percentage Problem Solving	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
		Solve problems that involve all fractions learning.	Solve problems that involve all fractions learning	Solve problems that involve all above fraction learning.	- Solve problems involving increasingly harder fractions to calculate quantities and fractions to divide quantities, including non-unit fractions where the answer is a whole number. - Solve simple, measure and money problems involving fractions and decimals to two decimal places.	Solve fraction and decimal problems involving numbers up to three decimal places. Solve problems that require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.	Solve fraction and decimal problems which require answers to be rounded to specified degrees of accuracy.

Ratio & Proportion

Ratio & Proportion

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Relative sizes. Percentage comparison, scale factors & unequal sharing							• Solve problems involving the relative sizes of two quantities, where missing values can be found by using integer multiplication and division facts. • Solve problems involving the calculation of percentages (for example, of measures, and such as 15% of 360) and the use of percentages for comparison. • Solve problems involving similar shapes where the scale factor is known or can be found. • Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples. • Solve problems involving ratio relationships.

Algebra

Fractions (including decimals and percentages)

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Fraction, Decimal and Percentage Problem Solving	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
		Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations and missing number problems such as $7 = ? - 9$	Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	Solve problems including missing number problems.			- Express missing number problems algebraically. - Use simple formulae (e.g. to find missing number, lengths, coordinates and angles) - Generate, describe and generalise linear number sequences. - Find pairs of numbers that satisfy an equation with two unknowns. - enumerate possibilities of combinations of two variables. - Solve problems with 2 unknowns.
		Although algebraic notation is not formally introduced until Year 6 algebraic thinking starts earlier, as exemplified by missing number problems.					

Measurement

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Comparing Measures	- Use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects. - Develop spatial reasoning skills across all areas of mathematics including shape, space and measures.	Compare and describe practical problems for; - Length and heights (e.g. long/short, longer/shorter, tall/short, double/half) - Mass/weight (e.g. heavy/light, heavier than, lighter than) - Capacity and Volume (e.g. full/empty, more than, less than, half, half full, quarter) - Time (e.g. quicker, slower, earlier, later) (moving from non-standard units to common standard units)	Compare and order lengths, mass, volume/capacity and record the results using $>$, $<$ and $=$ (e.g. half as high, twice as long)	Compare lengths (m/cm/mm) mass, volume/capacity (l/ml),	- Compare different measures, including money in pounds and pence. - Convert between different units of measure (e.g. kilometre to metre; hour to minute)		

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Measuring & Estimating Measures	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
		Measure and begin to record the following; - Length and heights - Mass/weight - Capacity and Volume - Time (hours, minutes, seconds) (beginning to use measuring tools such as a ruler, weighing scales and containers; moving from non-standard units to common standard units)	Choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels.	- Measure lengths (m/cm/mm) mass, volume/capacity (l/ml) including using mixed units (e.g. 1 kg and 200 g) - Divide 100 into 2, 4, 5 and 10 equal parts and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.	- Measure and record metric measures using decimal notations, including money in pounds and pence. - Estimate different measures, including money in pounds and pence. - measure	Divide 1 into 2, 4, 5 and 10 equal parts and read scales/number lines marked in units of 1, 2, 4, 5 and 10 equal parts.	Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Money Understanding & Recognition	• Use everyday language to talk about money.	• Recognise and know the value of different denominations of coins and notes.	• Recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular values. • Find different combinations of coins that equal the same amounts of money.	• Fluently use pounds (£) and pence (p) notation and understand the concept of giving change.	• Represent pounds and pence using decimal notations.		

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Tell the Time	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
		- Tell the time to the hour and half past the hour and draw the hands on a clock face to show these times. - Recognise and use language relating to dates, including days of the week, weeks, months and years.	Tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times.	- Tell the time from an analogue clock to the nearest minute, including using Roman Numerals from I to XII and 12 hour and 24-hour clocks. - Estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours such as o'clock/am/pm, morning, afternoon, noon and midnight.	Read, write and convert time between analogue and digital 12- and 24-hour clocks.		

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Comparing and Sequencing Events and Intervals		Sequence events in chronological order using language (e.g. before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening)	Compare and sequence intervals of time (e.g. what is longer - 40 minutes or half an hour?)	Compare durations of events (e.g. to calculate the time taken by two trains using a timetable and say which journey takes longer.			

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Problem Solving	Use everyday language to talk about size, weight, capacity, position, distance, time and money to solve problems.		Solve simple problems in a practical context involving addition and subtraction of money, including giving change.	- Add and subtract amounts of money to give change using both pounds (£) and pence (p) in practical contexts. - Add and subtract lengths in (m, cm, mm), mass (kg, g) and volumes (l/ml)	- Calculate different measures, including money in pounds and pence. - Solve problems involving converting from hours to minutes to seconds; years to months; weeks to days.	- Use all four operations to solve problems involving measures (money) using decimal notation, including scaling. - Use all four operations to solve problems involving measure (e.g. length, mass, volume/capacity, time) using decimal notation, including scaling. - Solve problems involving converting between units of time (e.g. how many seconds is 2.5 minutes?)	- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate. - Using a number line, use add and subtract positive and negative integers for measures such as temperatures.

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Conversion			Know the number of minutes in an hour and the number of hours in a day.	Know the number of seconds in a minute and the number of days in each month, year and leap year.	- Convert between different units of measurement (e.g. kilometre to metre, hour to minute) - Solve problems involving converting from hours to minutes, minutes to seconds, years to months, and weeks to days.	- Convert between units of metric measure (e.g. kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) including using common decimals and fractions. understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints.	- Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit and vice versa, using decimal notation of up to three decimal places. - Convert between mile and kilometres (including with a graphical representation).

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Area and Perimeter				Measure the perimeter of simple 2D shapes.	- Measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres. - Find the perimeter of regular and irregular polygons. - Find the area of rectilinear shapes by counting squares.	- Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres, including the relations of perimeter or area to find unknown lengths. - Calculate and compare the area of rectangles (including squares) and including using standard units, square centimetres (cm²) and square metre (m²) and estimate the area of irregular shapes.	- Recognise that shapes with the same areas can have different perimeters and vice versa. - Calculate the area of parallelograms and triangles, including the use of formulae (relating the area of rectangles and parallelograms and triangles, for example, by dissection and then calculate their areas). - Recognise when it is possible to use formulae for the area of shapes (e.g. for a triangle when the base and perpendicular height is known).

Measurement

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Volume						Estimate volume (e.g. using 1cm³ blocks to build cuboids, including cubes) and capacity (e.g. using water)	- Calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm³) and cubic metres (m³) and extending to other units (e.g. mm³ and km³) - Recognise when it is possible to use the formulae for the volume of shapes (e.g. cuboids)

Geometry - Properties & Shapes

Geometry – Properties & Shapes

*Black – National Curriculum Statutory Requirements

*Green – Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Identifying 2D Shapes & Their Properties	Explore characteristics of everyday objects and shapes and use mathematical language to describe them.	- Recognise and name common 2D shapes (e.g. rectangles (including squares), circles and triangles) in different orientations and sizes relating to everyday objects. - Recognise common 2D shapes presented in different orientations, and know that rectangles and triangles are not always similar to one another.	- Identify and describe the properties of 2D shapes (e.g. quadrilaterals and polygons), including the number of sides and line symmetry in a vertical line. - Use precise language to describe the properties of 2D and 3D shapes and compare shapes by reasoning about similarities and differences in properties.	Describe the properties of 2D shapes using accurate language, including lengths of lines and acute and obtuse angles, greater or lesser than a right angle.	- Identify regular polygons, including equilateral triangles and squares, as those in which the side lengths are equal and the angles are equal. - Identify lines of symmetry in 2D shapes presented in different orientations. - Reflect shapes in a line of symmetry and complete symmetrical figure or pattern with respect to a specified line of symmetry.	- Describe a range of geometric shapes, using conventional markings for parallel lines and right angles. - Use the properties of rectangles to deduce related facts and find missing lengths/angles. - Use the term diagonal and make conjectures about angles formed between sides and diagonals and parallel sides, and other properties of quadrilaterals.	- Describe the properties of geometric shapes and explain how unknown angles and lengths can be derived from known measurements. - Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius.

Geometry - Properties & Shapes

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Drawing & Constructing 2D Shapes		• Compose 2D shapes from smaller shapes to match an example, including manipulating shapes to place them in particular orientations.		• Draw 2D shapes (symmetrical and non-symmetrical polygons) • Draw polygons by joining marked points, and identify parallel and perpendicular sides.	• Complete a simple symmetrical figure with respect to a specific line of symmetry. • Draw polygons, specified by coordinates and translate within the first quadrant.	• Draw rectangles with sides accurate to the nearest mm.	• Draw, compose and decompose shapes according to given properties, including dimensions, angles and area, using conventional markings and labels for lines and angles and solve related problems.

Geometry – Properties & Shapes

*Black – National Curriculum Statutory Requirements

*Green – Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Comparing & Classifying 2D Shapes		• .	• Compare and sort common 2D shapes and everyday objects reasoning about similarities and differences in properties (e.g. sides and vertices).		• Compare and classify geometric shapes, including quadrilaterals (e.g. parallelogram, rhombus, trapezium) and triangles (e.g. isosceles, equilateral, scalene) based on their properties and sizes.	• Compare and classify a range of geometric shapes, using conventional markings for parallel lines and right angles. • Distinguish between regular and irregular polygons based on reasoning about equal sides and angles.	• Compare and classify a range of geometric shapes based on their properties and sizes, using conventional markings for parallel lines and right angles.

Geometry - Properties & Shapes

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Identifying 3D Shapes and the Properties	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
		- Recognise and name common 3D shapes (e.g. cuboids including cubes, pyramids and spheres) in different orientations and sizes, relating to everyday objects. - Recognise common 3D shapes presented in different orientations and know that cuboids and pyramids are not always similar to one another.	- Identify and describe properties of 3D shapes (e.g. cuboids, prisms and cones) including the number of edges, vertices and faces. - Use precise language to compare shapes by reasoning about similarities and differences in properties. - Identify 2D shapes on the surface of 3D shapes (e.g. a circle on a cylinder and a triangle on a pyramid).			Identify 3D shapes including cubes and other cuboids from 2D representations.	Identify and describe a range of 3D shapes.

Geometry - Properties & Shapes

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Drawing & Constructing 3D Shapes		• Compose 3D shapes from smaller shapes to match an example, including manipulating shapes to place them in particular orientations.		• Make 3D shapes using modelling ,materials; recognise 3D shapes in different orientations and describe them (symmetrical and non-symmetrical).			• Recognise, describe and build simple 3D shapes making nets.

Geometry - Properties & Shapes

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Comparing & Classifying 3D Shapes	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
			• Use precise language to describe the properties of common 3D shapes. • Compare and sort common 3D shapes and everyday objects, reasoning about similarities and differences in properties.				

Geometry – Properties & Shapes

*Black – National Curriculum Statutory Requirements

*Green – Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Angles				- Recognise that angles are a property of shape or a description of a turn. - Identify right angles in 2D shapes presented in different orientations. Recognise that two right angles make a half turn, three make three quarters of a turn and four make a complete turn; Identify whether angles are greater than or less than a right angle. - Identify horizontal, vertical lines and pairs of perpendicular and parallel lines.	Identify acute and obtuse angles and compare and order angles up to right angles by size.	- Draw given angles and measure them in degrees ($^{\circ}$) - Know angles are measured in degrees; estimate and compare acute, obtuse and reflex angles. Identify; - Angles at a point and one whole turn (total 360°) - Angles at a point on a straight line and $1/2$ a turn (total 180°) - Other multiples of 90° And use this understanding to deduce missing angles and solve angle problems.	- Recognise angles where they meet at a point, are on a straight line, or are vertically opposite and find missing angles. - Find unknown angles in any triangles, quadrilaterals and regular polygons.

Geometry - Position & Direction

Geometry - Position & Direction

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Patterns	• Recognise, create and describe patterns. • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed evenly.		• Order and arrange combinations of mathematical objects in patterns and sequences.				

Geometry - Position & Direction

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
Movement, Grids & Co-ordinates	• Use everyday language to talk about position and distance. • Develop spatial reasoning skills across all areas of mathematics including shape, space and measures.	• Describe position, directions and movement, including half, quarters and three-quarter turns.	• Use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise) and apply in a practical context (e.g. pupils themselves moving in turns, giving instructions to other pupils to do so and programming robots using instructions given in right angles)		• Describe the movements between positions as translations of a given unit to the left/right/and up/down. • Draw a pair of axes in one quadrant, with equal scales and integer labels. • Describe positions on a 2D grid as co-ordinates in the first quadrant (e.g. 2,5) • Plot specified points and draw sides to complete a given polygon. • Reflect shapes in a line of symmetry.	• Identify, describe, plot and read the position of a shape following a reflection (reflection should be in lines that are parallel to the axes) or translation in one quadrant grid, and know that the shape has not changed. • Describe positions on a 2D grid as co-ordinates in the first quadrant (e.g. 2,5) • Draw a pair of axes in one quadrant, with equal scales and integer labels.	• Draw and translate simple shapes in four quadrants and reflect them in the axes. • Draw a pair of axes in four quadrants with equal scales and integer labels. • Describe positions on the full co-ordinate grid (all four quadrants). • Draw and label quadrilaterals in four quadrants, predicting missing co-ordinates using the properties of shapes.

Statistics

Statistics

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Interpreting, Constructing and Solving Problems with Charts, Graphs & Tables	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
			- Interpret and construct simple pictograms, tally charts, block diagrams and simple tables (e.g. construct pictograms using many to one correspondence in pictograms with simple ratios 2, 5, 10). - Ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity. - Ask and answer questions about totalling and comparing categorical data.	- Interpret and present data using bar charts, pictograms and tables (including understanding and using simple scales (e.g. 2, 5, 10 units per cm) with increasing accuracy. - Solve one-step and two-step questions (e.g. How many more? And How many fewer?) using information presented in scaled bar charts, pictograms and tables.	- Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs. - Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.	- Solve comparison, sum and difference problems using information presented in a line graph. - Complete, read and interpret information in table, including timetables.	Interpret and construct pie charts (including deducing required angles of sectors by applying understanding of angles and fractions) and line graphs and use these to solve problems (including connecting conversion from kilometres to miles in measurements to its graphical representations)

Statistics

*Black - National Curriculum Statutory Requirements

*Green - Non-statutory 'Ready to Progress' criteria

Mean Average	<u>Reception</u>	<u>Year One</u>	<u>Year Two</u>	<u>Year Three</u>	<u>Year Four</u>	<u>Year Five</u>	<u>Year Six</u>
							• Calculate and interpret the mean as an average; understand when finding the mean is appropriate.

Early Learning Goals for Mathematics

EYFS – Maths

As per the EYFS framework, we believe developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be;

- To count confidently.
- To develop a deep understanding of the numbers to 10.
- To understand the relationships between numbers to 10.
- To identify patterns within those numbers.

By providing frequent and varied opportunities to build and apply this understanding - such as using manipulatives and tens frames for organising counting. We will provide opportunities for children to develop a secure base of knowledge and vocabulary from which the mastery of mathematics is built. In addition, it is important that the curriculum includes rich opportunities for children to develop their spatial reasoning skills across all areas of mathematics, including shape, space and measures, look for patterns and relationships, spot connections, 'have a go', talk to adults and peers about what they notice, whilst not being afraid to make mistakes.

Teachers will ensure that they reflect the varied ways that young pupils learn in their curriculum delivery. These 'characteristics of effective teaching and learning' are;

- **Playing and Exploring** - Children investigate and experience things, and 'have a go.'
- **Active Learning** - Children concentrate and keep on trying if they encounter difficulties and enjoy achievements.
- **Creating and thinking critically** - children have and develop their own ideas, make links between ideas and develop strategies for doing things.

EYFS – Maths

Within the EYFS Framework, the 'specific area' of mathematics has two ELGs:

- **Number** – Have a deep understanding of number 10, including the composition of each number. Subitise (recognise quantities without counting) up to 5. Automatically recall (without reference to rhymes, counting or aids) number bonds to 1, including double facts.
- **Numerical Patterns** – Verbally count beyond 20, recognising the pattern of the counting system. Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity. Explore and represent patterns within numbers up to 10, including evens and odds and how quantities can be distributed equally.

Long Term Planning Mathematics

Reception Long Term Maths Plan

Autumn Term

Wk 1	Wk 2	Wk 3	Wk 4	Wk 5	Wk6 & Wk7	Wk 8	Wk 9	Wk 10	Wk 11	Wk 12	Wk 13	Wk 14
Number: Place Value: counting and ordering numbers 1-5			Matching, sorting and comparing groups Same/different, equal, more, less, greater, fewer		Making patterns Copy, continue and created repeating patterns	Number: Addition - Sorting into groups Introduce part/whole model practically		Subitising 1,2,3,4,5	Number: Addition One more within 5		Number: Subtraction One less within 5 (inc 0)	
<u>NUMBER</u> Children should have a deep understanding of the numbers 1-5 including the composition of numbers 1-5 and be able to represent the composition using a part whole model. Children can subitise numbers to 5. Children are beginning to automatically recall number bonds to 5. E.g. 4 buttons – I can see a group of 2 and another group of 2					<u>NUMERICAL PATTERNS</u> Children should be able to count to 10 independently, understanding the pattern of the counting system is always adding one more. Children should be able to count one to one correspondence to 5 and know the last number is the final total. Children should revisit 'all gone' and see the connection to zero. Children should be able to compare two groups using more than, less than and then same/ equal				<u>WIDER MATHS</u> Children should be able to use the language of capacity, size and mass e.g. big, little, large, small, tall, short. Children should be able to make a simple repeating pattern with at least 3 units and spot mistakes within a pattern. Children should recognise and describe some simple 2D shapes e.g. circle, triangle and square. Children should be able to use the language of time of day e.g. day, night, morning, afternoon, before, after, day, tomorrow			

Reception Long Term Maths Plan

Spring Term

Wk 1	Wk 2	Wk 3	Wk 4	Wk 5	Wk 6	Wk 7	Wk 8	Wk 9	Wk 10	Wk 11	Wk 12
Addition and Subtraction: Number to 5: introducing zero		Addition and Subtraction: Number bonds to 5		Place value: Numbers to 10: Counting 6, 7, 8	Place value: Numbers to 10: Counting 9 and 10	Addition and Subtraction: Addition to 10: combining two groups to find the whole		Addition and Subtraction: Addition to 10 Number bonds to 10- ten frame		Addition and Subtraction: Addition to 10 Number bonds to 10- part-whole mode	
<u>NUMBER</u> Children should have a deep understanding of the majority of the numbers 1-8 and using resources be able to represent the composition of numbers using a part whole model. Children can order some numbers applying their number knowledge. Children can confidently subitise a small number of objects and use familiar concept images e.g. tens frame, fingers etc. Children are beginning to automatically recall number bonds to 5.				<u>NUMERICAL PATTERNS</u> Children should be able to verbally count to 20, understanding the pattern of the counting system is always adding one more and understanding that when you count backwards it is one less. Children should be able to count one to one correspondence to 10 in different contexts and compare two quantities using the language of more/ greater or fewer than and same/ equal.				<u>WIDER MATHS</u> Children will be able to use the language of weight to make comparisons and identify the heaviest and lightest. Children can make pairs of objects and measure using non-standard units. Children can explore and manipulate 3D shapes. Children should be able to use the language of time of day e.g. day, night, morning, afternoon, before, after, day, tomorrow			

Reception Long Term Maths Plan

Summer Term

Wk 1	Wk 2	Wk 3	Wk 4	Wk 5	Wk 6	Wk 7	Wk 8	Wk 9	Wk 10	Wk 11	Wk 12	Wk 13	Wk 14
Place value: Numbers to 10: Comparing groups up to 10	Number and Place value: Numbers to 20 Counting to 20	Numerical patterns Halving and sharing/ doubling		Number: Addition and Subtraction Adding by counting on		Number: Addition and Subtraction Taking away by counting back		Numerical patterns: odds and evens (link to spatial reasoning)		Number and Place value: Counting and comparing amounts within 20 (link to measure)		Geometry: Exploring patterns simple / complex patterns (consolidate 2d and 3D shapes)	
<u>NUMBER</u> Children should have a deep understanding of the majority of the numbers 1-10 and using resources be able to represent the composition of numbers using a part whole model and tens frame. Children can order some numbers, including sequences and apply their number knowledge. Children can confidently subitise to 5 and use familiar concept images e.g. tens frame, fingers etc. Children are able to automatically recall number bonds to 5. Children are aware of how groups change by adding and taking objects away.						<u>NUMERICAL PATTERNS</u> Children should be able to verbally count to 20, understanding the pattern of the counting system is always adding one more and understanding that when you count backwards it is one less. Children should be able to count one to one correspondence to 20 in different contexts and compare two quantities using the language of more/ greater or fewer than and same/ equal. Children understand the ordinal number system.						<u>WIDER MATHS</u> Children can explore and manipulate 2D and 3D shapes. Children can explore combining and separating as well as fitting shapes together and breaking shapes apart. Children will be able to use the language of weight to make comparisons and identify the heaviest and lightest. Children can make pairs of objects and measure using non-standard units. Children should be able to use the language of time of day e.g. day, night, morning, afternoon, before, after, day, tomorrow	

Year One Long Term Maths Plan

Term 1	Place Value (within 10) Read and write numerals from 1 to 10 in numerals and words Given a number, identify one more and one less	Addition & Subtraction (within 10) (focus on + - =)		Geometry - Shape	Place Value (within 20) Read and write numerals from 10 to 20 in numerals and words Given a number, identify one more and one less	Addition and Subtraction (up to 20 - including crossing 10)
Term 2	Multiplication and Division (up to 20, in 2s)		Place Value to 50 (in 10s)	Multiplication and Division (10s up to 50)	Fractions (halves)	Geometry: Position & Direction
Term 3	Number & Place Value to 100	Fractions (quarters)	Money (including counting in 5s)	Time	Measurement Length & Height, Weight & Volume	Four Operations Recap
Ongoing themes embedded throughout the year						
Autumn Term Money - recognising coins with snack Time - o clock Counting - one more and one less with (1 to 9)			Spring Term Time - half past Money Counting - one more one less (within 20) and multiples of 10s.		Summer Term Money Counting - counting backwards (crossing 10 and within 100)	

Year Two / Three Long Term Maths Plan

Term 1	Place Value to 100 < > = 10 more , 10 less	Addition and Subtraction - Number Bonds Mental method idea		Multiplication & Division Symbols and arrays 2s (Y2) 5s 10s		Geometry: Properties of Shape	
Term 2	Place Value Y2 - 2 digit Y3 - 3 digit 10 more/10 less	Fractions Year 2 - thirds, halves and quarters Year 3 – tenths and addition and subtraction of fractions		Addition & Subtraction recap – Y2 - 2 digit numbers Y3 formal method Partitioning Perimeter		Money Place value and addition / subtraction	
Term 3	Multiplication and Division Y2 5s and 10s Y3 4s and 8s Pictograms		Money Multiplication and Division		Geometry Position & Direction Y3 acute and obtuse angles	Measurement - Length height volume ca capacity	Themed based consolidation

Ongoing themes embedded throughout the year

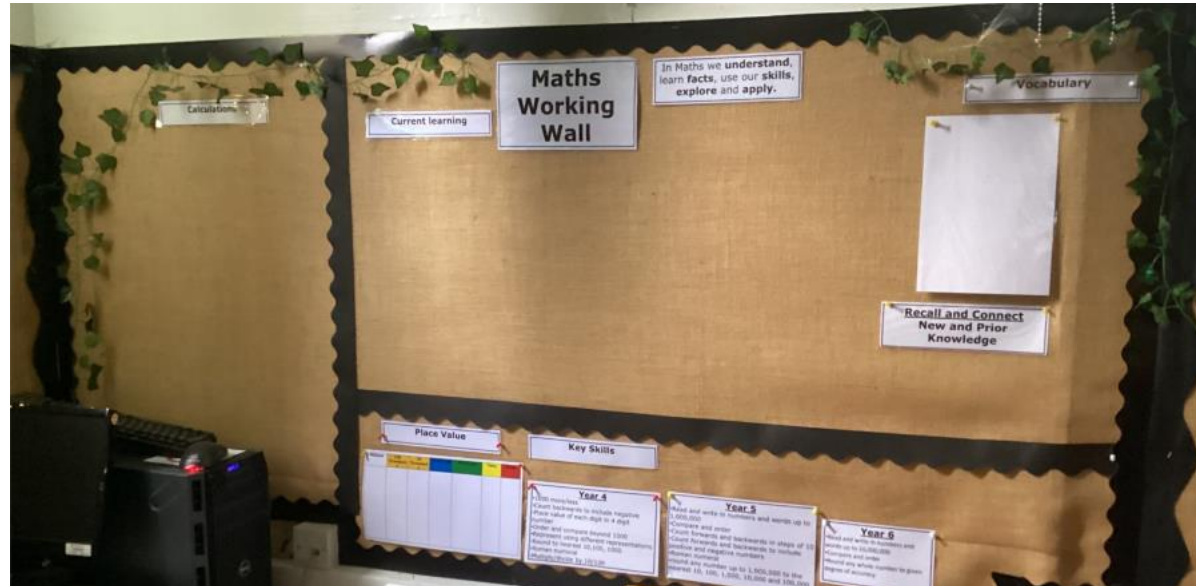
Autumn Counting in 2s 5s 10s (1 more 1 less) Money 2ps, 5ps and 10ps Length - 100cm Tally- house points	Spring Counting in 2s 3s and 5s Counting in 4s and 8s Bar charts	Summer Statistics marked and unmarked divisions
--	--	---

Year Four/Five/Six Long Term Maths Plan

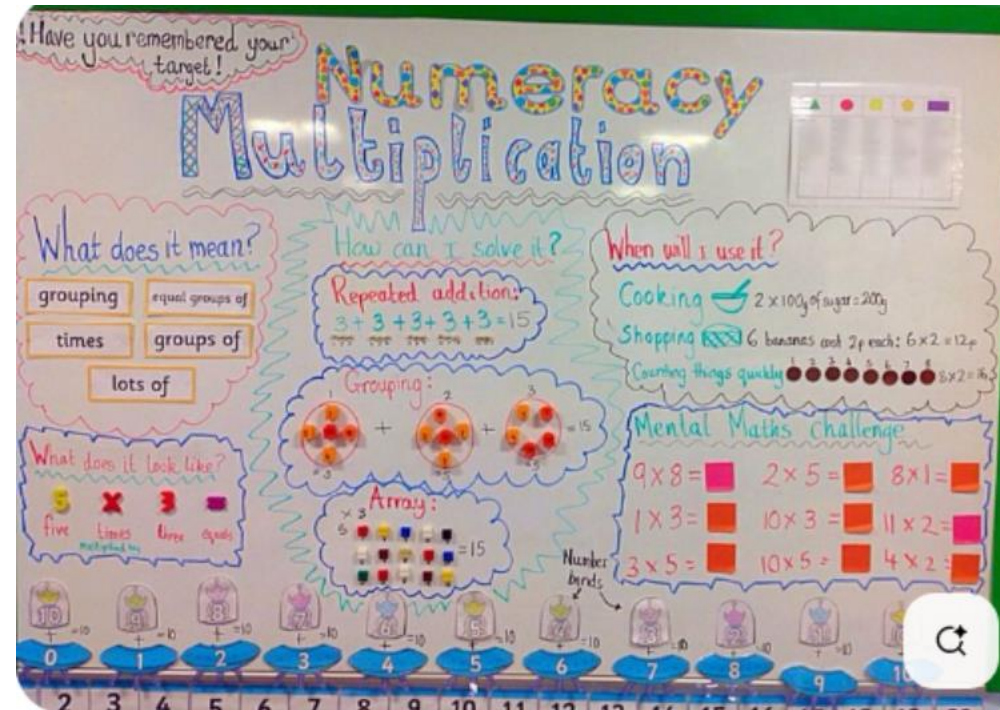
Term 1	Place Value Arithmetic focus place value addition/subtraction	Multiplication & Division Multiples and factors Formal written methods Y4 - 2s 5s 8s 10s 3s Area and volume Arithmetic: multiplication and division	Place Value Including rounding Arithmetic focus place value multiplication and division	Addition and Subtraction perimeter Arithmetic: addition and subtraction	
Term 2	Decimals and Percentages Y4 decimals tenths and hundredths - money and measurement 5/6 thousandths and percentages	Fractions Arithmetic focus - fractions		Geometry Shapes & Angles Types of triangles and Types of quadrilaterals Circles	
Term 3	Measurement Time including timetables Line graphs Y6 pie charts	Geometry Position & Direction Y4: Co-ordinates - Y5 and 6 Four quadrants and reflection/translations			Themed based Consolidation
Ongoing themes embedded throughout the year					
Autumn Counting in all Roman numerals as dates Attendance % Fractions - games, attendance		Spring Negative counting Common factors and multiples Money		Summer	

Working Wall Guidance

Maths working walls will follow the same presentation expectation with hessian and black border as well as include each of the sections shown below relevant to your year group. There will be a section to show calculations relevant to your year group, vocabulary and current learning. There will also be a place value grid with correct number of places for your year group and key skills to be displayed. A number will be written on each week and during times in the day they will complete key skills linked to that number.



Uniformed labels and place value grids are saved on Teams/General/Curriculum 2025/Displays/Working Walls/Maths



A display to show Multiplication Facts currently being taught. This will be different for each year group as in Key Stage one it will have the numbers you are counting. E.g. counting in 2s on socks. Lower Key Stage Two multiplication display to include the times table being taught and previous. In Upper Key Stage 2 all Times Tables to be displayed for children to reference. The display is to include facts, pictures, arrays and key vocab.

Concrete, Pictorial Abstract Approach

Concrete Pictorial Abstract - CPA

We know that our pupils learn best through a Concrete Pictorial Abstract approach (CPA). New learning is introduced using concrete and pictorial approaches, with rigorous support for children to move towards abstract methods. The initial concrete phase brings concepts to life, and can be characterised as the 'doing' stage. Manipulatives play a key role in this as they are the concrete resources used to support pupils thinking as they explore abstract ideas. Using something 'real' to make sense of the maths takes away the need to imagine or visualise at the early stage of learning a new concept. Concrete maths manipulatives provide the pupil a 'window' in, to make sense of the problem at hand by touching them, playing with them, exploring the patterns and relationships which make a huge difference between understanding for depth or just for procedure.

Time is taken to train and support teachers to use concrete objects correctly and effectively. Concrete objects are used where necessary, but it is also understood that after initial conceptual development has occurred, manipulatives are usually phased out, replaced by pictorial representations. This second stage of moving to the predominant use of pictorial resources can be characterised as the 'seeing' stage. Finally, pupils move to abstract methods – this can be characterised as the 'symbolic' stage. Here, concrete resources and pictorial representations are tools to supporting pupils to gain deep conceptual understanding, but should not be relied upon in the long term. However, it must be highlighted that at any point in a teaching sequence, pupils may move back and forth between concrete, pictorial and abstract representations, in order to support learning.

Concrete Pictorial Abstract - CPA

The amount of time pupils need to progress from concrete to abstract understanding varies. For some, it might occur within a single lesson, but for others, it will develop gradually over a teaching sequence. In each teaching sequence, pupils start with a concrete experience. As they move towards the pictorial, they might move between the pictorial and the concrete in order to clarify and refine their understanding, eventually reaching the abstract understanding.

Reliance and subsequent dependence on manipulatives and associated aids can hinder progression through the curriculum. Teachers need to give pupils enough time to consolidate learning and they need to plan for how pupils will move away from using the manipulatives. This will help to avoid pupils relying on manipulatives to work around gaps in core knowledge that might become barriers to learning later.

The concept of variation also applies to the use of concrete resources. Fundamental to maths mastery is the ability to use and apply concepts to a wide range of concepts by gaining deep transferable understanding of concepts. One step towards achieving this is through variation – variation of the problem posed, but also variation on the manipulatives used. Here, there is not just one concrete resource used to explore a concept and answer questions. For instance, when teaching number and place value, a range of manipulatives can be used, including: Numicon, bead strings, Dienes blocks, and place value counters. Pupils who require more support to master a specific concept often achieve success through the structured use of a wide variety of manipulatives.

.

Concrete Equipment

Bead Strings



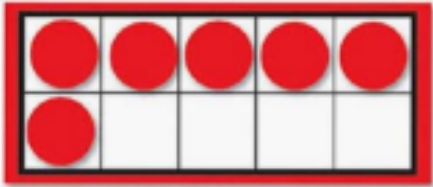
A class set of bead strings is an essential resource for classrooms. These consist of a short string with beads on, each alternate group of ten coloured differently. These are great for counting on and back in ones or ten, exploring number bonds to 10 or 100, and as a visual way of demonstrating fractions, decimals, percentages and the four operations. They are also an effective tool for looking at counting on and back, helping pupils understand number lines.

Cuisenaire Rods



Plastic or wooden rods are coloured coded depending on their size (from 1 unit to 10 units). They have a whole host of applications including number bonds, patterns, fractions, decimals, bar modelling, scaling and ratios.

Tens Frame



These support the central concept that a single digit also be partitioned and used to aid mental addition (or subtraction). Tens frames can also be used to teach fractions and decimals

Concrete Equipment

Dienes Apparatus



Plastic blocks, in ones, rows of ten and larger arrays of 100 and 1000s. They support pupils in developing an understanding of place value, as well as exploring the concept of regrouping in addition and subtraction, and modelling the process for long division.

Numicon



Structured apparatus with different sized and coloured pieces representing each number. Numicon's imagery uses patterns to represent each numeral. The patterns are structured so number relationships can be seen and experienced. These can also be used to teach about fractions, decimals and percentages. Great resources for small group or one-to-one intervention. Numicon is also weighted so you can use it with scales.

Geo boards



A great way for exploring geometry. Rubber bands are stretched over the pegs to make different shapes, perfect for exploring sides and vertices, investigating area and perimeter, symmetry and angles. A variety of different geoboards are available from isometric boards to coordinate boards.

Concrete Equipment

Place Value Counters



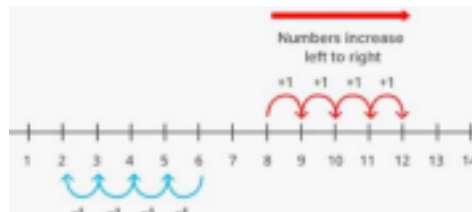
These counters show place value and are invaluable for teaching all four operations – particularly for multiplication and division. The counters give a concrete representation of the number, especially important when working with larger numbers and decimal numbers

Number Tracks



These are often used with younger pupils as a precursor to using number lines, for developing early understanding of number, counting on and counting back – can be used in conjunction with counters.

Number Lines



These are a very versatile resource and can be used to build understanding of all four operation

Concrete Equipment

Counters



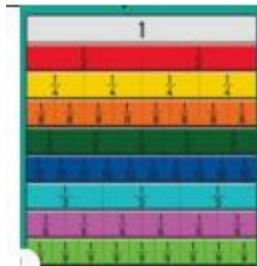
Counters are excellent for developing basic maths skills such as counting, sorting and pattern making.

Interlocking cubes



These are very useful for developing patterns, both one- and two-dimensional, based on colour. When the cubes are used to build three-dimensional structures, they lead naturally to the concepts of volume and surface area. They are also very useful for young pupils developing a wide range of basic number concepts.

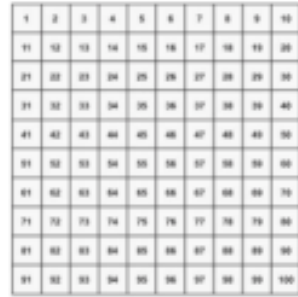
Fraction Boards



These are very useful visual representation to help pupils learn, compare and identify fractions, set out in the form of a wall. They are excellent for supporting both fraction equivalence and arithmetic.

Concrete Equipment

100 Squares



These can be used to support the development of many number concepts, such as number patterns, odd and even numbers, basic counting and the introduction of basic addition and subtraction.

Protractors



Protractors are a key resource for developing the skills of measuring and drawing angles.

2D shapes



Excellent for sorting, comparing and discussing the properties of 2-D shapes.

Concrete Equipment

100 Squares

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

These can be used to support the development of many number concepts, such as number patterns, odd and even numbers, basic counting and the introduction of basic addition and subtraction.

Protractors



Protractors are a key resource for developing the skills of measuring and drawing angles.

2D shapes



Excellent for sorting, comparing and discussing the properties of 2-D shapes.

Concrete Equipment

3d shapes



Excellent for sorting, comparing and discussing the properties of 3-D shapes.

Digit Cards



These are a very versatile resource for pupils to independently generate numbers when developing a wide range of number concepts.

Concrete **Pictorial** Abstract - Bar Modelling

Pictorial representations, the 'seeing' stage, are a key component of a pupil's transition from the use of concrete resources, when initially exploring a new concept, to gaining a deep conceptual understanding, where the related schema is internalised and only abstract 'symbolic' representation is required.

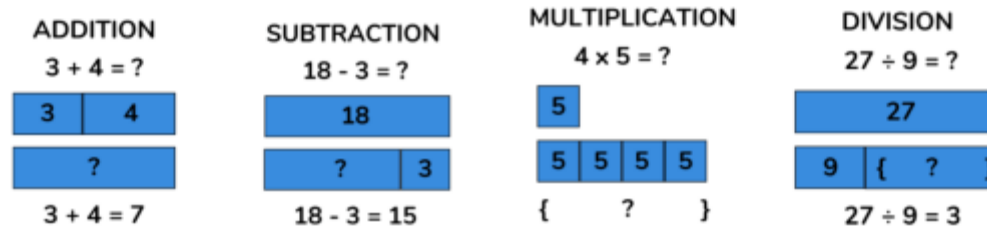
Bar models thus act as a 'bridge' between the concrete, pictorial and abstract (CPA in maths); once children are secure with using pictorial versions of their concrete materials, they can progress to using bars as visual representations.

The bar model is a pictorial representation of a problem or concept where bars or boxes are used to represent the known and unknown quantities. In word problems, bar models help children decide which operations to use and to visualise problems.

The bar model is integral to this pictorial approach when teaching number, and has a wide range of uses when modelling and exploring concepts across both key stage 1 and 2. Its value in supporting a deep understanding of underlying concepts is most apparent when tackling mastery questions that require reasoning and problem solving skills.

Concrete Pictorial Abstract - Bar Modelling

Bar models do not, however, do calculations for the pupil nor solve problems and reason by themselves; rather, they simply make it easier for pupils to understand the maths that is underpinning a question by providing a pictorial model, work out which calculation must be done and solve a problem accordingly.



Initially, when younger pupils are set a problem, real objects may be used to represent a problem. An example of a simple addition problem is: 'There are 5 green apples and 3 red apples in the bag. How many apples are in the bag? Here, a bag of real apples is initially used. The teacher pulls the apples out the bag, sorts them into red and green ones, puts them into a line and supports the pupils to count them. Here, parts are put together to find the whole

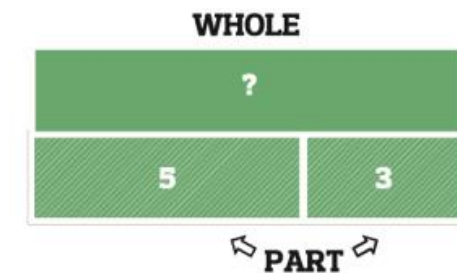


Concrete **Pictorial** Abstract - Bar Modelling

The next step in conceptual development, still remaining in the zone of concrete representation, is to model the problem using something else to represent the apples, such as green and red counters or cubes. Although this is still concrete, it is a generic representation. The objects can still be lined up and counted, but the objects could represent three and five of anything. 103. The representation of the problem now develops to using pictorial methods. One-to-one representation is still used, but now the objects are drawn. Actual apples could initially be drawn but pupils would usually settle comfortably on using simple picture such as a square or cross to represent each apple:

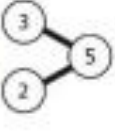
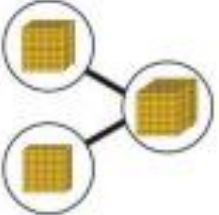
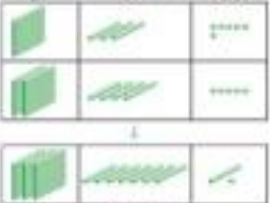
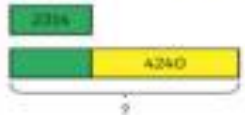
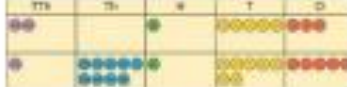


The teacher now progresses beyond one-to-one representation, using a proportional bar instead. Older pupils will usually grasp this conceptual leap quicker than younger pupils. One bar can be drawn to represent the whole, and the parts can be drawn underneath in a separate bar. More complex problems require different arrangements of bars, but the underlying principle is the same:

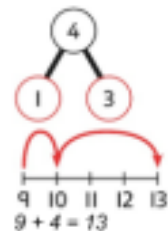


Calculation Policy

Addition

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6																																																																																																																																																															
Addition of 1- and 2-digit numbers to 20 Use numbered number lines and practical apparatus to add by counting on in ones. Encourage children to start with the larger number and count on: $6 + 3 = 9$  $5 + 7 = 12$ Put your finger on number eight and count on ten.    Use Numicon, bead strings and tens frames to introduce addition by bridging through ten:   7 add 3 makes 10. So, 7 add 5 is 10 and 2 more.	Addition of numbers with up to 2 digits Continue using number lines where required: $28 + 6 = 34$  Move onto blank number lines - 1. First counting on in 10s and 1s. 2. Become more efficient by adding the 1s in one jump. 3. Followed by adding both the 10s and the 1s in one jump: $34 + 23 = 57$  Also use blank number lines in conjunction with 100-squares to jump in 10s: $28 + 30 = 58$  Use known bonds to add 10s:  I know that $4 + 3 = 7$. So, I know that 4 tens add 3 tens is 7 tens.	Addition of numbers with up to 3 digits <u>Adding 100s:</u> Use known facts and unitising to add multiples of 100.  $3 + 4 = 7$ 3 hundreds + 4 hundreds = 7 hundreds $300 + 400 = 700$  Use the part-whole model to support abstract understanding of this:   $3 + 2 = 5$ $300 + 200 = 500$	Addition of numbers with up to 4 digits Reinforce the column method by using concrete methods, including base ten equipment, starting with no regrouping and then moving onto regrouping:  136 + 345 = 481 Find the sum of 2314 and 4240.  	Addition of numbers with more than 4 digits Represent additions using place value equipment on a place value grid alongside written methods.  <table><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>2</td><td>3</td><td>1</td><td>4</td><td>0</td></tr><tr><td>+</td><td>4</td><td>2</td><td>4</td><td>0</td></tr><tr><td colspan="5"> </td></tr><tr><td>6</td><td>5</td><td>5</td><td>4</td><td>0</td></tr></table> Use the compact method: 67454+4239= <table><tr><td>6</td><td>7</td><td>4</td><td>5</td><td>4</td></tr><tr><td>+</td><td>4</td><td>2</td><td>3</td><td>9</td></tr><tr><td colspan="5"> </td></tr><tr><td>6</td><td>1</td><td>6</td><td>8</td><td>3</td></tr><tr><td>1</td><td>1</td><td></td><td></td><td></td></tr></table> Introduce decimal addition through the use of base ten equipment, representing exchange where necessary.  <table><tr><th>O</th><th>Ts</th><th>Hs</th><th>Ts</th><th>Hs</th></tr><tr><td>1</td><td>2</td><td>0</td><td>5</td><td>7</td></tr><tr><td>+</td><td>2</td><td>0</td><td>5</td><td>5</td></tr><tr><td colspan="5"> </td></tr><tr><td>1</td><td>4</td><td>1</td><td>1</td><td>3</td></tr></table>	TTh	Th	H	T	O	2	3	1	4	0	+	4	2	4	0						6	5	5	4	0	6	7	4	5	4	+	4	2	3	9						6	1	6	8	3	1	1				O	Ts	Hs	Ts	Hs	1	2	0	5	7	+	2	0	5	5						1	4	1	1	3	Addition of numbers of increasing complexity and decimal value Use the compact method, illustrating the addition of more complex decimals with differing numbers of decimal places: <table><tr><td>2</td><td>3</td><td>.</td><td>3</td><td>6</td><td>1</td></tr><tr><td>9</td><td>.</td><td>0</td><td>8</td><td>0</td><td></td></tr><tr><td>5</td><td>9</td><td>.</td><td>7</td><td>7</td><td>0</td></tr><tr><td>+</td><td>1</td><td>.</td><td>3</td><td>0</td><td>0</td></tr><tr><td colspan="6"> </td></tr><tr><td>9</td><td>3</td><td>.</td><td>5</td><td>1</td><td>1</td></tr><tr><td>2</td><td>1</td><td>.</td><td>2</td><td></td><td></td></tr></table> Practice adding several numbers with more than 4 digits: <table><tr><td>8</td><td>1</td><td>.</td><td>0</td><td>5</td><td>9</td></tr><tr><td>3</td><td>.</td><td>6</td><td>6</td><td>8</td><td></td></tr><tr><td>1</td><td>5</td><td>.</td><td>3</td><td>0</td><td>1</td></tr><tr><td>+</td><td>2</td><td>0</td><td>5</td><td>5</td><td>1</td></tr><tr><td colspan="6"> </td></tr><tr><td>1</td><td>2</td><td>0</td><td>5</td><td>7</td><td>9</td></tr><tr><td>1</td><td>1</td><td>.</td><td>1</td><td></td><td></td></tr></table>	2	3	.	3	6	1	9	.	0	8	0		5	9	.	7	7	0	+	1	.	3	0	0							9	3	.	5	1	1	2	1	.	2			8	1	.	0	5	9	3	.	6	6	8		1	5	.	3	0	1	+	2	0	5	5	1							1	2	0	5	7	9	1	1	.	1		
TTh	Th	H	T	O																																																																																																																																																																
2	3	1	4	0																																																																																																																																																																
+	4	2	4	0																																																																																																																																																																
6	5	5	4	0																																																																																																																																																																
6	7	4	5	4																																																																																																																																																																
+	4	2	3	9																																																																																																																																																																
6	1	6	8	3																																																																																																																																																																
1	1																																																																																																																																																																			
O	Ts	Hs	Ts	Hs																																																																																																																																																																
1	2	0	5	7																																																																																																																																																																
+	2	0	5	5																																																																																																																																																																
1	4	1	1	3																																																																																																																																																																
2	3	.	3	6	1																																																																																																																																																															
9	.	0	8	0																																																																																																																																																																
5	9	.	7	7	0																																																																																																																																																															
+	1	.	3	0	0																																																																																																																																																															
9	3	.	5	1	1																																																																																																																																																															
2	1	.	2																																																																																																																																																																	
8	1	.	0	5	9																																																																																																																																																															
3	.	6	6	8																																																																																																																																																																
1	5	.	3	0	1																																																																																																																																																															
+	2	0	5	5	1																																																																																																																																																															
1	2	0	5	7	9																																																																																																																																																															
1	1	.	1																																																																																																																																																																	

Support bridging through 10 with accompanying part-whole models:



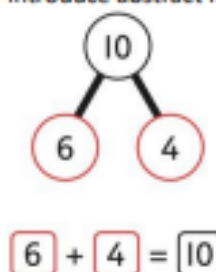
Introduce regrouping into 10s and 1s:



Introduce the bar model:



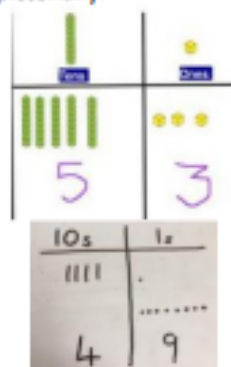
Use the part-whole model to introduce abstract methods:



Use a range of resources and pictorial methods, including bar modelling:



Use base ten apparatus to build understanding of 2-digit places value. Move onto drawing the rods pictorially:



Move onto expanded method as an intermediary to the compact method if required:

$$\begin{array}{r} 42 = 40 + 2 \\ + 36 = 30 + 6 \\ \hline 70 + 8 = 78 \end{array}$$

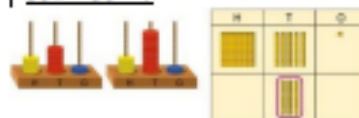
Culminate in compact method, supported by base ten apparatus where required:

Adding with no exchanging or bridging using concrete objects, base ten apparatus and number lines. Progress from adding 1s to adding 2/3 digit numbers:



$$245 + 4 = 249$$

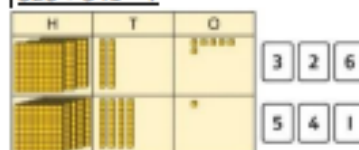
$$351 + 30 = ?$$



$$5 \text{ tens} + 3 \text{ tens} = 8 \text{ tens}$$

$$351 + 30 = 381$$

$$326 + 541 = ?$$



Reinforce exchanging with a range of place value equipment, including where there are multiple exchanges:

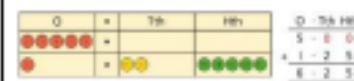


Use expanded method as an intermediary to compact method if required, illustrating place value:

$$\begin{array}{r} 4256 \\ + 1987 \\ \hline 6243 \end{array}$$

add ones
add tens
add hundreds
add thousands

Include examples where the numbers of decimal places are different.



Apply this to numbers with different numbers of decimal places using abstract column method:

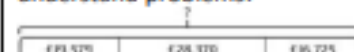
$$£23.59 + £4.38 =$$

$$\begin{array}{r} £ \quad 23.59 \\ + \quad 4.38 \\ \hline £ \quad 27.97 \end{array}$$

Teach how to align decimal places correctly:

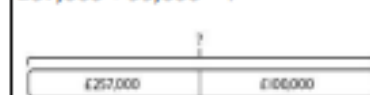
$$\begin{array}{r} 19.01 \\ 3.65 \\ + 0.70 \\ \hline 23.36 \end{array}$$

Use bar models to represent and understand problems:



Use a bar model to support thinking in addition problems.

$$257,000 + 99,000 = ?$$



I added 100 thousands then subtracted 1 thousand.

$$257 \text{ thousands} + 100 \text{ thousands} = 357 \text{ thousands}$$

$$257,000 + 100,000 = 357,000$$

$$357,000 - 1,000 = 356,000$$

$$\text{So, } 257,000 + 99,000 = 356,000$$

Represent and use number bonds and related subtraction facts within 20

Break apart a group and put back together to find and form number bonds.



$$3 + 4 = 7$$

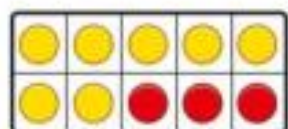


$$8 = 2 + 4$$

Use five and ten frames to represent key number bonds.

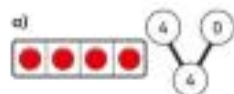


$$5 = 4 + 1$$



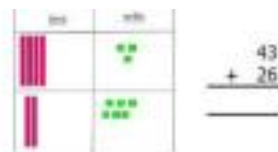
$$10 = 7 + 3$$

Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero.



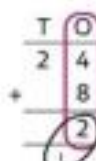
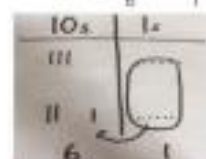
$$4 + 0 = 4$$

$$3 + 1 = 4$$

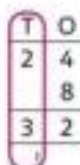


Use both concrete and pictorial methods to for regrouping, focusing on base ten equipment.

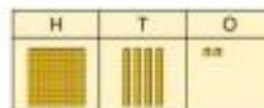
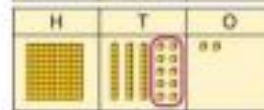
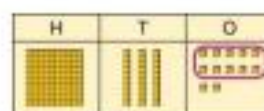
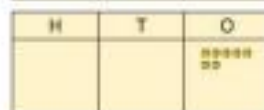
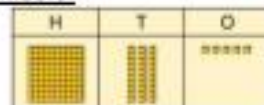
$$36 + 25$$



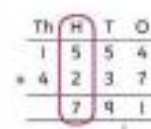
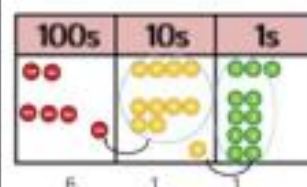
Culminate in the compact abstract method for regrouping, supported by base ten apparatus where required:



Adding with exchanging, using base ten equipment - concrete and pictorial methods. Begin with adding 1s and then adding 2/3 digit numbers:

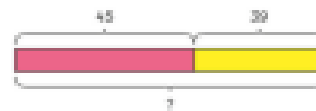
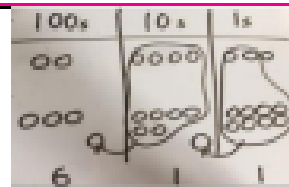


$$135 + 7 = 142$$



Culminate with compact method, supported by base ten equipment where required. Include examples that exchange in more than one column

Apply this method to money and measures.



Use expanded method prior to the compact method as an additional step if required:

$$\begin{array}{r} 275 \\ + 185 \\ \hline 120 \\ 140 \\ 300 \\ \hline 452 \end{array}$$

Culminate in compact method, supported by base ten equipment where required:

$$\begin{array}{r} \text{H T O} \\ 275 \\ + 185 \\ \hline \end{array}$$

$$\begin{array}{r} \text{H T O} \\ 275 \\ + 185 \\ \hline 91 \end{array}$$

$$\begin{array}{r} \text{H T O} \\ 275 \\ + 185 \\ \hline 291 \end{array}$$

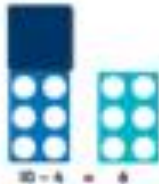
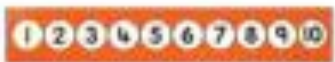
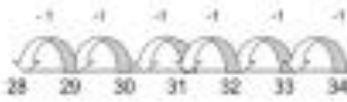
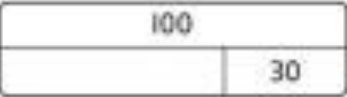
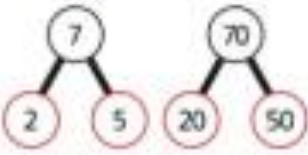
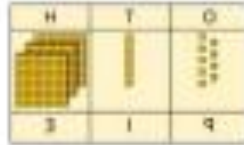
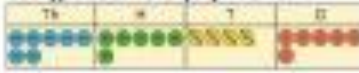
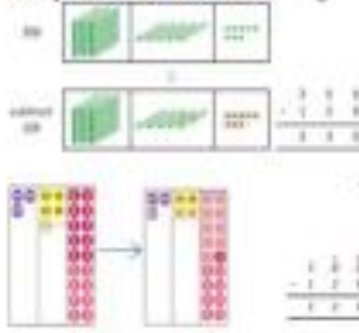
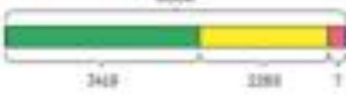
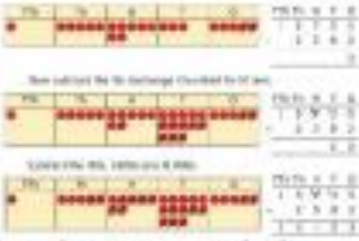
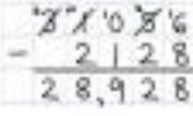
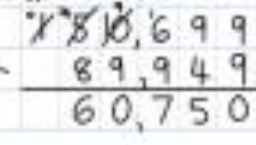
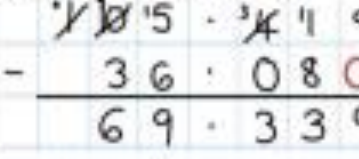
$$\begin{array}{r} \text{H T O} \\ 126 \\ + 217 \\ \hline 343 \end{array}$$

$$\begin{array}{r} \text{H T O} \\ 126 \\ + 217 \\ \hline 343 \end{array}$$

$$\begin{array}{r} \text{H T O} \\ 126 \\ + 217 \\ \hline 343 \end{array}$$

This should include where multiple exchanging is needed.

Subtraction

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<u>Subtraction of 1- and 2-digit numbers to 20</u> Consolidate understanding of subtraction using concrete and pictorial methods, showing subtraction on bead strings, Numicon, cubes and number tracks:  $10 - 4 = 6$  $10 - 4 = 6$  $9 - 5 = 4$ Put your finger on number nine. Count back five. Children draw and cross out or use counters to represent objects from a problem.  $9 - 4 = 5$ There are 5 children left.	<u>Subtraction of numbers with up to 2 digits</u> Continue counting back in 1s and 10s on number lines: $34 - 6 = 28$  $58 - 30 = 28$  Use known bonds to subtract multiples of 10:  8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.  $10 - 3 = 7$. So, 10 tens subtract 3 tens is 7 tens. 	<u>Subtraction of numbers with up to 3 digits</u> <u>Subtracting 100s:</u> Use known facts a to subtract multiples of 100 using CPA approaches:  $4 - 2 = 2$ $400 - 200 = 200$  Use a range of concrete methods, and then pictorial methods to represent subtraction with 3 digits, including base ten equipment, bar modelling and the part-whole model. Start with no regrouping/subtracting 1s and move onto regrouping/subtracting 3-digit numbers:  $319 - 4 = ?$	<u>Subtraction of numbers with up to 4 digits</u> Support use of mental methods where appropriate, using base 10 equipment:  $7,646 - 40 = 7,606$ Reinforce the column method by using concrete methods:  Use pictorial methods, including bar modelling:  Move onto expanded method as an intermediary to compact method if required, illustrating place value:	<u>Subtraction of numbers with more than 4 digits</u> Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. $15,735 - 2,582 = 13,153$  Use the compact method:  Use a place value grid and pictorial methods to support subtraction of decimals, including exchanging: 	<u>Subtraction of numbers of increasing complexity and decimal values</u> Use the compact method with larger numbers:  Illustrate the subtraction of more complex decimals with differing numbers of decimal places: 

Progress to a marked number line:

$$12 - 9 = 3$$



Use the part-whole model to support abstract methods:

There are 9 bean bags altogether. How many bean bags does Ruby have on her head?



Use the part-whole method to introduce regrouping into 10s and 1s:



Step 2: Subtract 3 from 7.
 $7 - 3 = 4$

Step 3: $10 - 3 = 7$

Peter has 4 toy cars left.

Subtract 10s and 1s with the support of ten frames also.

For example: $18 - 12$

Use ten frames to represent the efficient method of subtracting 12.

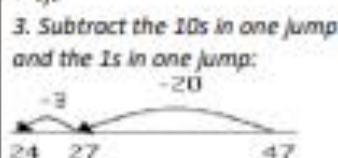


First subtract the 10, then subtract 2.

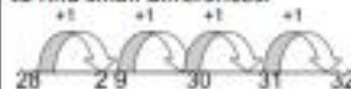
Use complimentary addition to introduce the idea of differences, using small differences only:

Continue using blank number lines with increasingly more efficient jumps:

1. First count back in 10s & 1s.
2. Become more efficient by subtracting the 1s in one jump (by using the known fact $7 - 3 = 4$).
3. Subtract the 10s in one jump and the 1s in one jump:

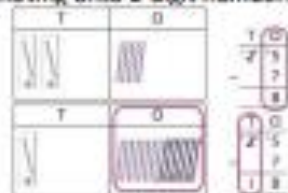


Continue using number lines to find small differences:

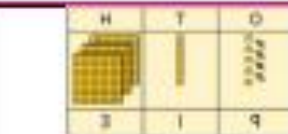
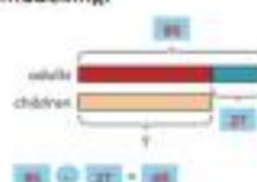


The difference between 28 and 32 is 4.

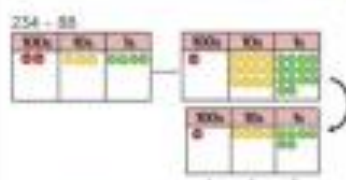
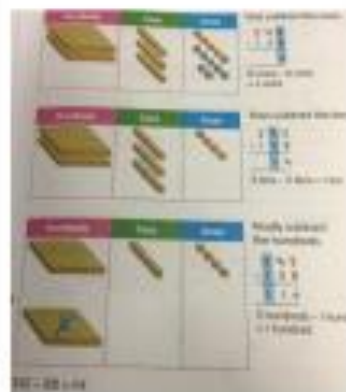
Subtract single-digit numbers using exchanging, before moving onto 2-digit numbers:



Use a range of concrete and pictorial methods, including bar modelling:



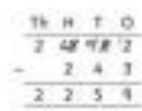
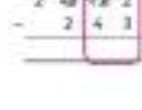
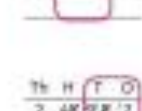
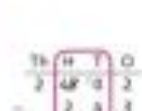
$$9 - 4 = 5 \quad 319 - 4 = 315$$



Use bar models to represent subtractions. Find the difference is represented as two bars for comparison.



Culminate with compact method, including where there are multiple exchanges:



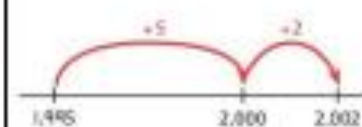
Apply this method to money and measures.

Apply column method to numbers with different numbers of decimal places:

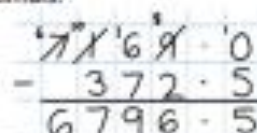


To subtract two large numbers that are close, children find the difference by counting on:

$$2,002 - 1,995 = ?$$



Teach how to align decimal places correctly and to add place value holders, including with decimals:

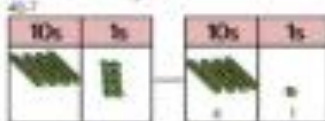


$11 - 9 = 2$



The difference between nine and eleven is two.

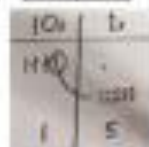
Use base ten apparatus to build understanding of 2-digit place value. Move onto drawing the rods pictorially:



Move onto expanded method as an intermediary to the compact method with no regrouping if required:

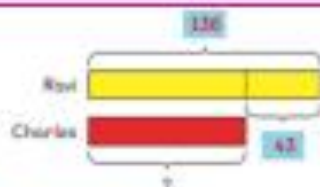
$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} \quad \begin{array}{r} 80 + 9 \\ - 50 + 7 \\ \hline 30 + 2 = 32 \end{array}$$

Use concrete methods, and then pictorial methods for regrouping:



Culminate in the abstract compact method for regrouping, supported by base ten equipment where required:

$$\begin{array}{r} 89 \\ - 57 \\ \hline 32 \end{array}$$



$$136 - 43 = 93$$

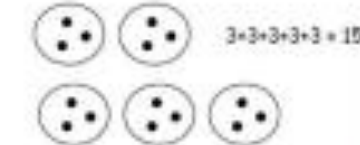
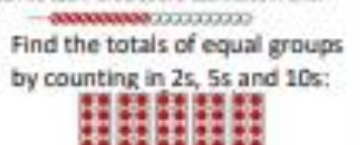
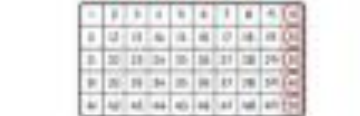
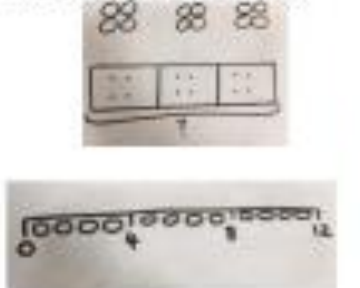
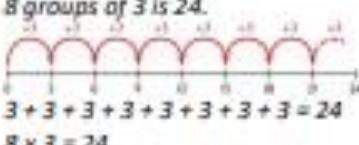
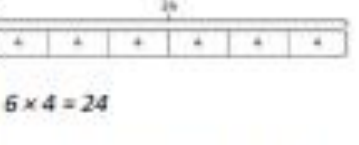
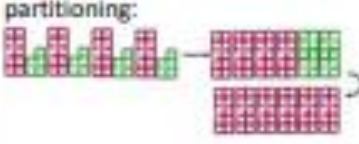
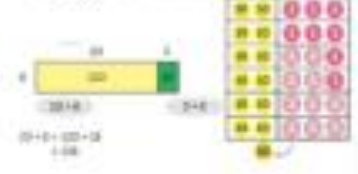
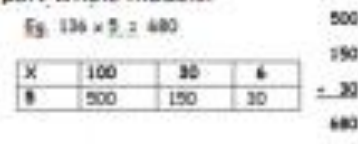
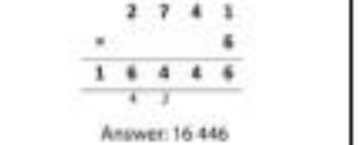
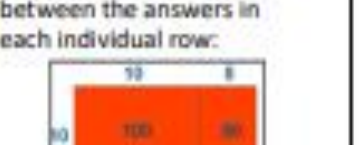
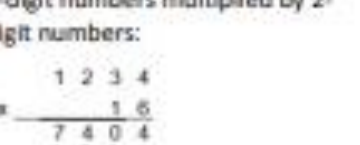
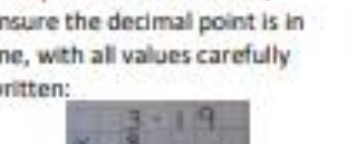


391	
186	?

Culminate with abstract compact method, supported by base ten equipment where required:

$$\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$$

Multiplication

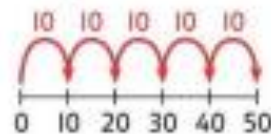
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<u>Solve one-step problems involving multiplication</u> Use a variety of concrete methods and pictorial methods (e.g. bead strings, cubes, real objects), to multiply by grouping (including the use of repeated addition): How many legs will 3 rabbits have?  $2 \times 4 = 8$ How many legs will 3 rabbits have?  $2 \times 4 = 8$ There are 3 sweets in one bag. How many sweets are in 5 bags altogether?  $3 \times 3 = 3 + 3 + 3 = 9$ Can we count forwards and backwards in tens?  Find the totals of equal groups by counting in 2s, 5s and 10s: 	<u>Calculate mathematical statements for multiplication and within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs</u> Use arrays with concrete methods to multiply two 1-digit numbers, including to show commutativity: $2 \times 5 = 5 \times 2$  $2 \times 5 = 10$ $5 \times 2 = 10$ Employ the use of variety of pictorial and abstract methods, including the bar model, 100-square, bead strings, number lines and repeated addition: 	<u>Multiply two-digit numbers by one-digit numbers</u> Cement understanding of link between repeated addition and multiplication. 8 groups of 3 is 24.  $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24$ $8 \times 3 = 24$ A bar model may represent multiplications as equal groups.  $6 \times 4 = 24$ Use concrete methods such as Numicon, base ten apparatus or place value counters to multiply by partitioning:  Concrete methods can be replaced with pictorial methods for partitioning, including the part-whole model: $23 \times 3 = ?$	<u>Multiply two-digit and three-digit numbers by a one-digit number</u> Use a variety of CPA methods to develop understanding of multiplication calculations:  Use partitioning in multiplication through the use of the grid method and part-whole models: Eg. $136 \times 5 = 680$  Use expanded short multiplication as a precursor to short multiplication if required: $127 \times 6 = 762$ 	<u>Multiply numbers up to 4 digits by a 1- or 2-digit number, including long multiplication for 2-digit numbers</u> Continue developing the grid method for 1-digit multiplied by 4-digit numbers: $6 \times 2000 = 12000$ $6 \times 700 = 4200$ $6 \times 40 = 240$ $6 \times 1 = 6$ Move onto short multiplication: 2741×6 becomes  Answer: 16446 Introduce the grid method for long multiplication to illustrate the relationship between the answers in each individual row:  $20 \times 12 = (20 \times 10) + (20 \times 2) = 220$ 	<u>Multiply numbers up to 4 digits by a 2-digit number, including long multiplication for 2-digit numbers</u> The same as year 5, progressing to long multiplication once pupils are secure with grid method: 24×16 becomes  Answer: 384 Introduce long multiplication for 4-digit numbers multiplied by 2-digit numbers:  When applying short multiplication to decimals, ensure the decimal point is in line, with all values carefully written:  Here, briefly return to grid method for multiplying decimals if required.

Use arrays to support early multiplication:



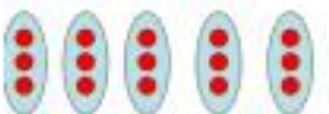
The group of ten faces - how many faces altogether? 2, 4, 5, 8, 10
Two groups of ten faces - how many faces altogether? 5, 10

Use a number line to support repeated addition through counting in 2s, 5s and 10s:



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

$$5 \times 2 = 5 + 5 = 10$$



$$5 \text{ groups of } 3 = 5 \text{ lots of } 3 = 3 + 3 + 3 + 3 + 3 = 15$$

$$5 \text{ times } 3 = 3 \text{ multiplied by } 5 = 5 \times 3 = 15 = 3 \times 5 = 15$$

Use pictorial methods to support the calculation of mathematical statements:



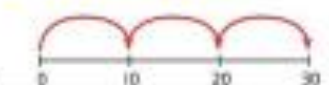
$$3 \times 4 = 12$$



Learning the 2x, 5x & 10x tables

Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.

$$10 + 10 + 10 = 30 \quad 3 \times 10 = 30$$

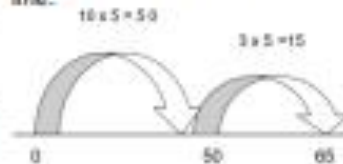


10s	1s
00	000
00	000
00	000
6	9

$$12 \times 4 = 48$$

$$10 \times 4 = 40 \quad 2 \times 4 = 8$$

Partitioning can also be demonstrated on a number line:



Use place value to support how partitioning is linked with multiplying by a 2-digit number.

$$3 \times 24 = ?$$

T	O
20	20
20	20
20	20

$$3 \times 4 = 12$$

T	O
20	20
20	20
20	20

$$3 \times 20 = 60$$

$$60 + 12 = 72$$

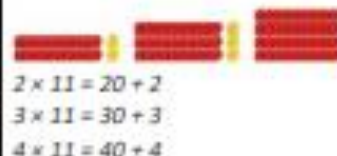
$$3 \times 24 = 72$$

Culminate in short multiplication:

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \end{array}$$

Understanding all times-tables up to 12×12

Represent the relationship that the 9, 11 and 12 times tables have with the 10 times table.



Understand how times-tables relate to counting patterns

Understand links between the $\times 3$ table, $\times 6$ table and $\times 9$ table:

$$5 \times 6 \text{ is double } 5 \times 3$$

$$\times 5 \text{ table and } \times 6 \text{ table:}$$

$$I \text{ know that } 7 \times 5 = 35 \text{ so I know that } 7 \times 6 = 35 + 7.$$

$$\times 5 \text{ table and } \times 7 \text{ table:}$$

$$3 \times 7 = 3 \times 5 + 3 \times 2$$

Use expanded long multiplication if pupils have difficulty transitioning straight to vertical long multiplication:

$$23 \times 13 = 299$$

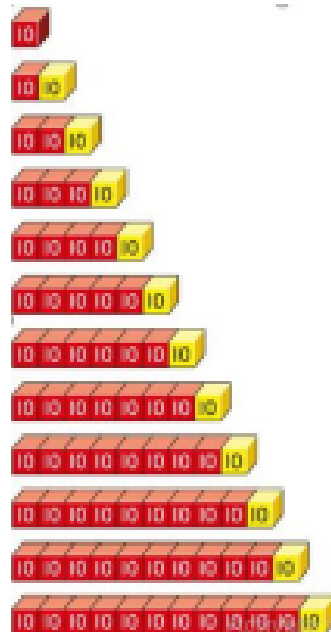
$$\begin{array}{r} 23 \\ \times 13 \\ \hline 69 \quad (3 \times 3) \\ 230 \quad (3 \times 20) \\ \hline 299 \quad (10 \times 3) \\ 230 \quad (10 \times 20) \\ \hline 299 \end{array}$$

Culminate in vertical long multiplication once pupils are secure, including multiplying 3- and 4-digit numbers by 2-digit numbers:

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \\ + 1120 \\ \hline 1512 \end{array}$$

If pupils are making errors, move back to expanded long multiplication or grid method as appropriate.

Understand how the times-tables increase and contain patterns.



Introduce the abstract grid method, using partitioning:

$$\begin{array}{r} 10 \quad 4 \\ 6 \quad \boxed{60 \quad 24} \\ 60 + 24 = 84 \end{array}$$

Expanded column method may be used as a bridge to compact method, but children must understand the link with place value and exchange.

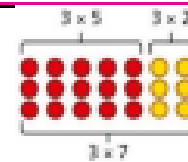
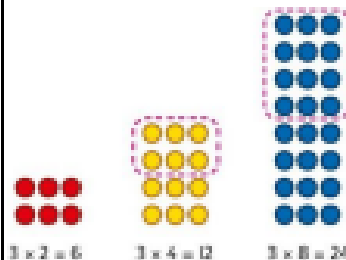
T	O	T	O
_____	_____	1	5
_____	_____	x	6
_____	_____	+	_____
_____	_____		6 x 5
_____	_____		6 x 10

Culminate in short multiplication:

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 1 \quad 8 \quad 8 \\ \times \quad 2 \quad 4 \\ \hline 7 \quad 9 \quad 2 \end{array}$$

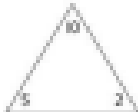
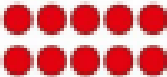
Understanding and using times tables

Children understand how the x2, x4 and x8 tables are related through repeated doubling.

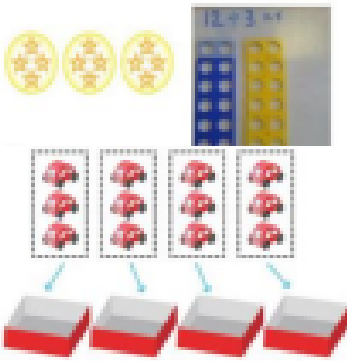
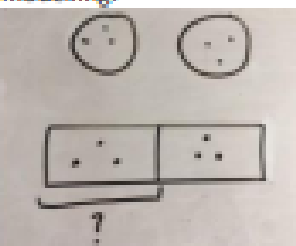
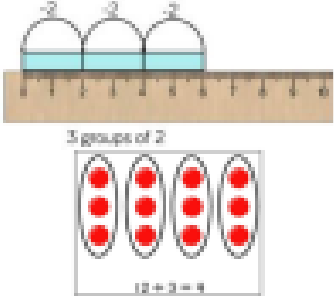
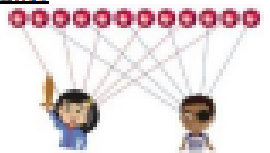
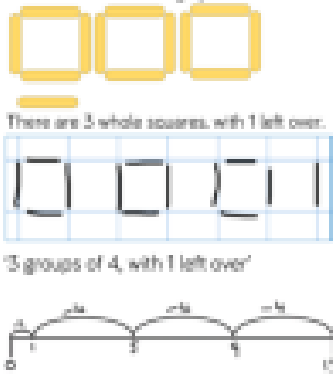
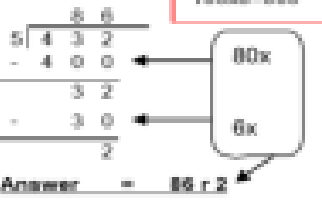
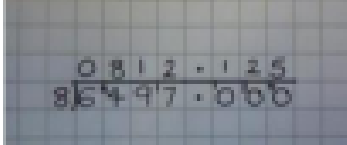


x9 table and x10 table:

$$6 \times 10 = 60 \quad 6 \times 9 = 60 - 6$$

		Children understand the relationship between related multiplication and division facts in known times-tables. $2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 5 = 2$ $10 \div 2 = 5$  			
--	--	--	--	--	--

Division

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<u>Solve one-step problems involving division</u> Make equal groups using a variety of concrete methods first, including finding a half and a quarter by grouping into 2 and 4: How many groups of 4 can be made with 12 stars? 3  Share these eight apples equally between two children. How many apples will each child have?  Represent grouping and sharing using pictorial methods, including bar modelling: 	<u>Calculate mathematical statements for division within the multiplication tables and write them using the multiplication ($\times$), division ($\div$) and equals ($=$) signs</u> Continue using a range of concrete methods to group objects, including the concrete representation of repeated subtraction, arrays of objects and bead strings:  $15 \div 3 = 5$ Children should understand this number sentence as 'How many groups of 3 make 15?' And on a bead string  <u>Develop division as sharing equally:</u> 	<u>Calculate mathematical statements for division, progressing to short division</u> Introduce 2-digit numbers divided by 1-digit numbers with remainders using the CPA approach, starting with practical resources such as lollipop sticks or Cuisenaire rods:  There are 3 whole squares, with 1 left over. '3 groups of 4, with 1 left over' Before introducing the formal method short division, introduce sharing using place value counters: $42 \div 3 = 14$ 	<u>Calculate mathematical statements, including short division for 3-digit numbers divided by 1-digit numbers</u> Divide multiples of 10 and 100 by a single digit: $9 \div 3 = 3$ $90 \div 3 = 30$ $900 \div 3 = 300$ $9 \div 3 = 3$ <i>9 tens divided by 3 is 3 tens.</i> <i>9 hundreds divided by 3 is 3 hundreds.</i> Support division of 2- and 3- digit numbers through the use of partitioning, using pictorial and abstract methods: $39 \div 3 = ?$  $39 = 30 + 9$ $30 \div 3 = 10$ $9 \div 3 = 3$ $39 \div 3 = 13$	<u>Divide numbers up to 4 digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context</u> Continue using chunking to divide 3-digit numbers by 1-digit numbers, beginning to subtract larger amounts of the divisor each time. Write a useful facts box on the side to support this. Remainders need to be shown as a whole number, or rounded up or down depending on context: $9 \div 5$  Answer = $1 \text{ r } 4$ Move onto chunking with 4-digit numbers: 	<u>Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context</u> Continue using the methods taught in year 5, culminating in the consistent use of short division, including the representation of the remainder as a decimal, according to context. Here, the number of decimal places required is added to the divisor, and the 'carrying over' continues beyond the decimal point:  Also use short division, interpreting the remainder as a fraction: $496 \div 11$ becomes  Answer: $45 \frac{1}{11}$

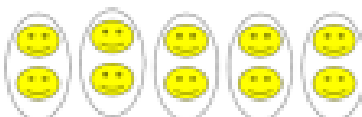
Children may relate this to counting back in steps of 2, 5 or 10:



Include the use of arrays to support early division:



How many faces altogether? How many groups of two?



Five groups of two

12 shared equally between 2. They get 6 each.

Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared.



15



They get 5 each.

Represent the objects shared into equal parts using a bar model

20 shared into 5 equal parts. There are 4 in each part.



Use a bar model to support understanding of division as sharing.



$$18 \div 2 = 9$$

Show the written method of 'chunking', using division facts to take chunks away, relating this to short division:

$$\begin{array}{r} 12 \\ 3 \overline{) 36} \\ \underline{- 30} \\ 6 \\ \underline{- 6} \\ 0 \end{array}$$

Move children onto the formal written method of short division once full understanding of previous methods is secure. Initially use questions with no calculation of remainders in any stage of the calculation:

Remind children of correct place value, that 98 is equal to 90 and 8, but in short division, pose:

$$\begin{array}{r} 32 \\ 3 \overline{) 98} \end{array}$$

- How many 3s in 9? = 3, and record it above the 9 tens.
- How many 3s in 8? = 2, and record it above the 8 units.

Move onto questions containing remainders within the calculation but not the answer – pupils need to 'carry over':

$$\begin{array}{r} 18 \\ 4 \overline{) 72} \end{array}$$

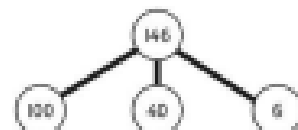
When solving problems, visual representations such as bar modelling can still be used to support:



$$36 \div 2 = 18$$

Sum has 18 sweets.

$$142 \div 2 = ?$$



$$100 \div 2 = \square \quad 40 \div 2 = \square \quad 2 \div 2 = \square$$

$$100 \div 2 = 50$$

$$40 \div 2 = 20$$

$$2 \div 2 = 1$$

$$50 + 20 + 1 = 71$$

$$142 \div 2 = 71$$

Continue to build understanding using chunking – the abstract written method which is the precursor to short division:

$$\begin{array}{r} 100 \\ 2 \overline{) 200} \\ \underline{- 200} \\ 0 \end{array}$$

$$\begin{array}{r} 100 \\ 2 \overline{) 200} \\ \underline{- 200} \\ 0 \end{array}$$

$$75 \div 6 = 12 \text{ remainder } 3$$

quotient

Move onto the same chunking method with 3-digit numbers, moving onto remainders if appropriate:

$$\begin{array}{r} 100 \\ 2 \overline{) 200} \\ \underline{- 200} \\ 0 \end{array}$$

→ 100 tens
→ 100 ones
→ remainder

Culminate in short division of 3-digit numbers by 1-digit numbers, with no remainders, but moving beyond this with remainders if appropriate:

Progress to short division of 4-digit numbers by 1-digit numbers, with remainders. Note that division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it – i.e. as a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem:

$$\begin{array}{r} 0663r5 \\ 8 \overline{) 5309} \end{array}$$

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Introduce dividing up to 4 digits by a two-digit whole number using chunking in the first instance, continuing to use a useful facts box. Start with simple chunks (x10, 100 etc), and then move onto more efficient chunks:

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

Answer : 27

Interpret long division answers as fractions and as decimals, according to context:

$$\begin{array}{r} 38 \\ 15 \overline{) 570} \\ \underline{- 450} \\ 120 \\ \underline{- 120} \\ 0 \end{array}$$

Answer: 38

$$\frac{38}{15} = \frac{76}{30}$$

$$38 \div 15 = 2 \frac{8}{15}$$

$$38 \div 15 = 2.533$$

Develop concept of division as grouping:

Understand how to make equal groups from a whole:



8 divided into 4 equal groups.
There are 2 in each group

Understand the relationship between grouping and the division statements:

$$12 \div 3 = 4$$



$$12 \div 4 = 3$$



$$12 \div 6 = 2$$



$$12 \div 2 = 6$$



Continue using arrays to support early division:

$$15 \div 5 = 3$$

$$15 \div 3 = 5$$



How many groups of 3?
How many groups of 5?
15 shared between 3 people is...?
15 shared between 5 people is...?

$$15 \div 3 = 5$$

$$15 \div 5 = 3$$

Repeated subtraction can also be represented using abstract number lines:
 $30 \div 6$



Using times tables to divide

Use knowledge of known times-tables to calculate divisions:



48 divided into groups of 4.
There are 12 groups.

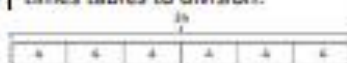
$$4 \times 12 = 48$$

$$48 \div 4 = 12$$



$$48 \div 4 = 12$$

Use bar models and number lines to support application of times tables to division:

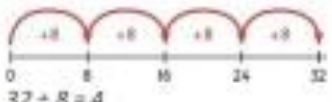


$$24 \div 4 = 6$$

$$24 \div 6 = 4$$



$$24 \div 8 = 3$$



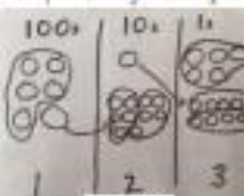
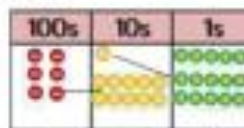
$$32 \div 8 = 4$$

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$$

$$\begin{array}{r} 037 \\ 5 \overline{) 185} \end{array}$$

Continue using a CPA approach for dividing 3-digit numbers by 1-digit numbers, to support understanding of division with some pupils where required:

$$615 \div 5$$



$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Understand the relationship between division and multiplication using times tables

Use objects to explore families of multiplication and division facts:



$$4 \times 6 = 24$$

24 is 6 groups of 4.

24 is 4 groups of 6.

24 divided by 6 is 4.

24 divided by 4 is 6.

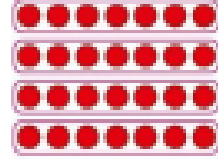
Represent divisions using an array:

Record as mathematical statements once secure:

$$15 \div 5 = 3$$

Associate the law of commutativity to show link between multiplication and division:

$$\begin{array}{l} 5 \times 2 = 10 \quad \text{—————} \quad 10 \div 2 = 5 \\ 2 \times 5 = 10 \quad \text{—————} \quad 10 \div 5 = 2 \end{array}$$



$$20 \div 4 = 5$$

Understand families of related multiplication and division facts:

I know that $5 \times 7 = 35$ so I know all these facts:

$$5 \times 7 = 35$$

$$7 \times 5 = 35$$

$$35 = 5 \times 7$$

$$35 = 7 \times 5$$

$$35 \div 5 = 7$$

$$35 \div 7 = 5$$

$$7 = 35 \div 5$$

$$5 = 35 \div 7$$

Developing Mathematical Language

Mathematical Vocabulary

Language is essential to the development of mathematical skills. Without a firm grasp of the vocabulary surrounding the subject of mathematics, things can quickly become disjointed and confusing to young minds.

The following spoken language activities should be undertaken with pupils to support the development of mathematical vocabulary.

- **Explaining** – giving a clear and detailed account;
- **Describing** – putting observations and experiences into words;
- **Categorising** – classifying according to common characteristics;
- **Making connections** – between items or information;
- **Interpreting** – perceiving the significance of connections;
- **Predicting** – using available information to estimate outcomes;
- **Comparing** – observing similarities and differences between items and relationships;
- **Contrasting** – observing differences between items or relationships;
- **Clarifying** – making clear their understanding;
- **Justifying** – providing evidence to prove a point;
- **Elaborating** – developing and extending ideas;
- **Planning** – organising ideas, stating ways of proceeding;
- **Raising new questions** – children ask their own questions and present issues;
- **Investigating** – examining systematically in order to solve a problem;
- **Evaluating** – judging and assessing;
- **Arguing/conceding a case** – presenting opposing/supporting reasons to a statement;
- **Reasoning** – drawing conclusions from facts and evidence;
- **Hypothesising** – suggesting an explanation for a group of facts;
- **Reciting** – reading or chanting aloud;
- **Recounting** – sharing personal experience and findings;

Reception Mathematical Vocabulary

Mathematical Vocabulary Progression

General

Reception

- **Describe:** When the curriculum asks pupils to 'describe' a mathematical object, transformation or the features of a graph, or anything else of a mathematical nature, it is asking pupils to refine their skills to home in on the essential mathematical features and to describe these as accurately and as succinctly as possible.
- **Hundred Square:** A 10 by 10 square grid numbered 1 to 100.
- **Number line:** A line where numbers are represented by points upon it.
- **Number track:** A numbered track along which counters might be moved. The number in a region represents the number of single moves from the start.
- **Pattern:** A systematic arrangement of numbers, shapes or other elements according to a rule: A series of numbers or other elements which follow a rule.

Mathematical Vocabulary Progression

Number and Place Value

Reception

- **Count:** The act of assigning one number name to each of a set of objects (or sounds or movements) in order to determine how many objects there are.
- **First:** Comes before all others in time or position.
- **Next:** Comes immediately after the present one in order.
- **Last:** Comes after all others in time or order. **Second:** an ordinal number, after first.
- **Zero:** Nought or nothing; zero is the only number that is neither positive nor negative

Mathematical Vocabulary Progression

Addition and Subtraction, Multiplication and Division

Reception

- **Addition:** The binary operation of addition on the set of all real numbers that adds one number of the set to another in the set to form a third number which is also in the set. The result of the addition is called the sum or total.
- **Altogether:** in total.
- **Difference:** In mathematics (as distinct from its everyday meaning), difference means the numerical difference between two numbers or sets of objects and is found by comparing the quantity of one set of objects with another.
- **Double:** 1) To multiply by 2. Example: Double 10 is $(10 \times 2) = 20$; 2) The number or quantity that is twice another. Example: 20 is double 10.
- **Equal:** Symbol: $=$, read as 'is equal to' or 'equals'. and meaning 'having the same value as'. Example: $7 - 2 = 4 + 1$ since both expressions, $7 - 2$ and $4 + 1$ have the same value, 5.
- **Fewer:** A lesser amount – used when counting discrete objects, i.e. countable objects such as, pens, teddies, counters, etc.
- **Less:** A smaller amount or not as much.
- **Minus:** A name for the symbol $-$, representing the operation of subtraction.
- **More:** A greater amount.
- **Number bond:** A pair of numbers with a particular total e.g. number bonds for ten are all pairs of whole numbers with the total 10.
- **Plus:** A name for the symbol $+$, representing the operation of addition
- **Share:** One model for the process of division (year R) Subtract: Carry out the process of subtraction (year R)

Mathematical Vocabulary Progression

Addition and Subtraction, Multiplication and Division

Reception

- **Subtract:** Carry out the process of subtraction.
- **Sum:** The result of one or more additions.
- **Take away:** Subtraction as reduction.
- **Total:** The sum found by adding.

Mathematical Vocabulary Progression

Fractions

Reception

- **Half:** One of two equal parts of a shape, quantity or object.

Mathematical Vocabulary Progression

Measurement

Reception

- **Before:** prior to
- **Clock:** a tool to measure time.
- **Take away:** Subtraction as reduction.
- **Total:** The sum found by adding.
- **Empty:** Containing nothing. Most commonly used in the context of measures.
- **Full:** Contains/holds as much or as many as possible; has no empty space.
- **Length:** A linear measurement.
- **Long:** An adjective used to describe length.
- **Pound (money):** Symbol £. A unit of money. £1.00 = 100 pence. £1 is commonly called a pound
- **Short:** An adjective used to describe length.
- **Tall:** Measuring a specific distance from top to bottom; opposite of short

Mathematical Vocabulary Progression

Geometry – Properties of Shape

Reception

- **Circle:** A common 2D shape with one curved side.
- **Corner:** A point where two or more lines or line segments meet. More correctly called vertex, vertices (plural). For example, a rectangle has four corners or vertices.
- **Cube:** In geometry, a three-dimensional figure with six identical, square faces. Adjoining edges and faces are at right angles.
- **Curved surface:** The curved boundary of a 3-D solid, for example; the curved surface of a cylinder between the two circular ends, or the curved surface of a cone between its circular base and its vertex, or the surface of a sphere.
- **Cuboid:** A three-dimensional figure with six rectangular faces.
- **Cylinder:** A three-dimensional object whose uniform cross-section is a circle.
- **Edge:** A line segment joining two vertices of a plane figure (2-D shape) and the intersection of two plane faces (in a 3-D shape).
- **Face:** One of the flat surfaces of a solid shape. Example: a cube has six faces; each face being a square.
- **Flat:** A level surface.
- **Oblong:** sometimes used to describe a non-square rectangle – i.e. a rectangle where one dimension is greater than the other.
- **Rectangle:** A parallelogram with an interior angle of 90° . Opposite sides are equal. If adjacent sides are also equal, the rectangle is sometimes referred to as an oblong.

Mathematical Vocabulary Progression

Geometry – Properties of Shape

Reception

- **Side:** A line segment that forms part of the boundary of a figure. Also edge.
- **Surface:** An outer boundary of a 3D object.
- **Square:** A quadrilateral with four equal sides and four right angles.
- **Triangle:** A polygon with three sides.

Mathematical Vocabulary Progression

Geometry – Position and Direction

Reception

- **Above:** Used to describe a higher position than another object.
- **Backwards:** a movement in the direction behind.
- **Below:** Used to describe a lower position than another object.
- **Between:** Indicates a position in relation to two other places or objects on either side
- **Direction:** The orientation of a line in space.
- **Forwards:** a movement in the direction in front

Key Stage One Mathematical Vocabulary

Mathematical Vocabulary Progression

General

Year One

- **Compare:** In mathematics when two entities (objects, shapes, curves, equations etc.) are compared one is looking for points of similarity and points of difference as far as mathematical properties are concerned.
- **Fluency:** To be mathematically fluent one must have a mix of conceptual understanding, procedural fluency and knowledge of facts to enable you to tackle problems appropriate to your stage of development confidently, accurately and efficiently.
- **Quantity:** Something that has a numerical value.
- **Symbol:** A letter, numeral or other mark that represents a number, an operation or mathematical idea.

Year Two

- **Consecutive:** Following in order. Consecutive numbers are adjacent in a count.
- **Counter example:** Where a general statement is offered, an example that clearly disproves it.
- **General statement:** A statement that applies correctly to relevant cases e.g. the sum of two odd numbers is an even number.
- **Generalise:** To formulate a general rule.
- **Relationship:** The way in which two or more things are connected.
- **Scale:** A measuring device usually consisting of points on a line with equal intervals.

Mathematical Vocabulary Progression

Number & Place Value

Year One

- **Cardinal number:** A cardinal number denotes quantity, as opposed to position within a series. 1, 2 and 3 are cardinal numbers. 1st, 2nd & 3rd are ordinal.
- **Decreasing:** Becoming smaller in value.
- **Digit:** One of the symbols of a number system most commonly the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Examples: the number 29 is a 2-digit number; there are three digits in 2.95. The position or place of a digit in a number conveys its value.
- **Even number:** An integer that is divisible by 2.
- **Increasing:** Becoming greater in value. Used in relation to number Sequences.
- **Numeral:** A symbol used to denote a number. The Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used in the Hindu-Arabic system giving numbers in the form that is widely used today.
- **Odd number:** An integer that has a remainder of 1 when divided by 2.
- **Ordinal number:** A term that describes a position within an ordered set. Example: first, second, third.
- **Partition:** To split a number into component parts. Example: $38 = 30 + 8$ or $19 + 19$.
- **Place value:** The value of a digit that relates to its position or place in a number
- **Rule:** A procedure for carrying out a process. In the context of patterns and sequences a rule, expressed in words or algebraically, summarises the pattern or sequence and can be used to extend it.

Year Two

- **Sequence:** A succession of terms formed according to a rule.

Mathematical Vocabulary Progression

Addition, Subtraction, Multiplication and Division

Year One

- **Array:** An ordered collection of counters, numbers etc. in rows and columns.
- **Divide:** To carry out the operation of division.
- **Division:** An operation on numbers interpreted in a number of ways. Division can be sharing – the number to be divided is shared equally into the stated number of parts; or grouping – the number of groups of a given size is found. Division is the inverse operation to multiplication
- **Facts:** Multiplication / division/ addition/ subtraction facts. The word 'fact' is related to the four operations and the instant recall of knowledge about the composition of a number. i.e. an addition fact for 20 could be $10+10$; a subtraction fact for 20 could be $20-9=11$. A multiplication fact for 20 could be 4×5 and a division fact for 20 could be $20 \div 5 = 4$
- **Missing number problems:** A problem of the type $7 = \square - 9$ often used as an introduction to algebra.
- **Number sentence:** A mathematical sentence involving numbers. Examples: $3 + 6 = 9$ and $9 > 3$.
- **Sign:** A symbol used to denote an operation. Examples: addition sign $+$, subtraction sign $-$, multiplication sign $\times$, division sign $\div$, equals sign $=$ etc. In the case of directed numbers, the positive $+$ or negative $-$ sign indicates the direction in which the number is located from the origin along the number line.

Year Two

- **Calculate:** To compute or work out mathematically
- **Commutative:** A property of addition and multiplication. It does not matter in which order the addends or factors are added or multiplied; the result will be the same.
- **Consecutive:** Following in order. 2, 3, 4, 5, 6 are consecutive numbers. 3, 6 and 9 are consecutive multiples of 3.
- **Dividend:** In division, the number that is divided.
- **Four operations:** Common shorthand for the four arithmetic operations of addition, subtraction, multiplication and division.
- **Greater than:** An inequality between numbers. The symbol used to represent greater than is an arrow pointing towards the smallest number.
- **Inequality:** When one number, or quantity, is not equal to another. Statements such as $a \neq b$, $a < b$ or $a \geq b$ are inequalities.
- **Inverse operations:** Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. $5 + 6 - 6 = 5$. Multiplication and division are inverse operations e.g. $6 \times 10 \div 10 = 6$.
- **Less than:** An inequality between numbers. The symbol used to represent less than is an arrow pointing towards the smallest number.
- **Multiple:** for any whole number, another number is a multiple if it is the answer to a multiplication question with the first number (e.g. 14, 49 and 70 are all multiples of 7)

Mathematical Vocabulary Progression

Addition, Subtraction, Multiplication and Division

Year One

Year Two

- **Multiplication:** Multiplication (often denoted by the symbol "x") is the mathematical operation of scaling one number by another. It is one of the four binary operations in arithmetic (the others being addition, subtraction and division).
- **Multiply:** Carry out the process of multiplication.
- **Repeated addition:** The process of repeatedly adding the same number or amount. One model for multiplication. Example $5 + 5 + 5 + 5 = 5 \times 4$.
- **Repeated subtraction:** The process of repeatedly subtracting the same number or amount. One model for division. Example $35 - 5 - 5 - 5 - 5 - 5 - 5 - 5 = 0$ so $35 \div 5 = 7$ remainder 0

Mathematical Vocabulary Progression

Fractions

Year One

- **Fraction:** The result of dividing one integer by a second integer, which must be non- zero. The dividend is the numerator and the non-zero divisor is the denominator.
- **Half:** one of two equal parts of a number, shape or quantity.
- **Quarter:** One of four equal parts of a whole, quantity or object.

Year Two

- **Denominator:** In the notation of common fractions, the number written below the line i.e. the divisor.
- **Equivalent fractions:** Fractions with the same value as another. For example: $\frac{4}{8}$, $\frac{5}{10}$, $\frac{8}{16}$ are all equivalent fractions and all are equal to $\frac{1}{2}$.
- **Non-unit Fraction:** A fraction with a numerator greater than one.
- **Numerator:** In the notation of common fractions, the number written on the top – the dividend (the part that is divided). In the fraction $\frac{2}{3}$, the numerator is 2
- **Simple fraction:** A fraction where the numerator and denominator are both integers. Also known as common fraction or vulgar fraction
- **Unit fraction:** A fraction that has 1 as the numerator and whose denominator is a non-zero integer. Example: $\frac{1}{2}$, $\frac{1}{3}$

Mathematical Vocabulary Progression

Measurement

Year One

- **Analogue Clock:** A clock usually with 12 equal divisions labelled 'clockwise' from the top 12, 1, 2, 3 and so on up to 11 to represent hours. Commonly, each of the twelve divisions is further subdivided into five equal parts providing sixty minor divisions to represent minutes. The clock has two hands that rotate about the centre. The minute hand completes one revolution in one hour, whilst the hour hand completes one revolution in 12 hours.
- **Capacity:** the volume of a material (typically liquid or air) held in a vessel or container. Note: the term 'volume' is used as a general measure of 3-dimensional space and cannot always be used as synonymously with capacity.
- **Hour:** A unit of time. One twenty-fourth of a day. 1 hour = 60 minutes = 3600 (60 x 60) seconds.
- **Length:** The extent of a line segment between two points. Length is independent of the orientation of the line segment.
- **Minute:** Unit of time. One-sixtieth of an hour. 1 minute = 60 seconds.
- **Scale:** An object used to measure mass
- **Second:** A unit of time. One-sixtieth of a minute.
- **Temperature:** A measure of the hotness of a body, measured by a thermometer or other form of heat sensor
- **Time:** 1. Progress from past, to present and to future 2. Time of day, in hours, minutes and seconds; clocks and associated vocabulary 3. Duration and associated vocabulary 4. Calendar time in days, weeks, months, years 5. Associated vocabulary such as later, earlier, sooner, when, interval of time, clock today, yesterday, tomorrow, days of the week, the 12 months of a year, morning, a.m., afternoon, p.m., noon, etc

Year Two

- **Centimetre (cm):** A unit of linear measure equivalent to one hundredth of a metre.
- **Chronological:** Relating to events that occur in a time ordered sequence.
- **Gram:** Symbol: g. The unit of mass equal to one thousandth of a kilogram.
- **Litre:** Symbol: l. A metric unit used for measuring volume or capacity. A litre is equivalent to 1000 cm³

Mathematical Vocabulary Progression

Measurement

Year One

- **Volume:** A measure of three-dimensional space. Usually measured in cubic units; for example, cubic centimetres (cm³) and cubic metres (m³)
- **Weight:** In everyday English weight is often confused with mass. In mathematics, and physics, the weight of a body is the force exerted on the body by the gravity of the earth, or any other gravitational body

Year Two

Mathematical Vocabulary Progression

Geometry – Properties of Shape

Year One

- **2-D/3-D:** Short for 2-dimensional and 3-dimensional. A figure is two-dimensional if it lies in a plane. A solid is three-dimensional and occupies space.
- **Polygon:** A closed plane figure bounded by straight lines. The name derives from many angles.
- **Pyramid:** A solid with a polygon as the base and one other vertex, the apex, in another plane. Each vertex of the base is joined to the apex by an edge. Other faces are triangles that meet at the apex. Pyramids are named according to the base: a triangular pyramid (which is also called a tetrahedron, having four faces), a square pyramid, a pentagonal pyramid etc.
- **Sphere:** A closed surface, in three-dimensional space, consisting of all the points that are a given distance from a fixed point, the centre. A hemisphere is a half-sphere

Year Two

- **Axis of symmetry:** A line about which a geometrical figure, or shape, is symmetrical or about which a geometrical shape or figure is reflected in order to produce a symmetrical shape or picture. Reflective symmetry exists when for every point on one side of the line there is another point (its image) on the other side of the line which is the same perpendicular distance from the line as the initial point.
- **Base:** The line or face on which a shape is standing.
- **Octagon:** A polygon with eight sides.
- **Pentagon:** A polygon with five sides and five interior angles.
- **Prism:** A solid bounded by two congruent polygons that are parallel (the bases) and parallelograms (lateral faces) formed by joining the corresponding vertices of the polygons. Prisms are named according to the base e.g. triangular prism, quadrangular prism, pentagonal prism.
- **Quadrilateral:** A polygon with four sides.
- **Symmetry:** A plane figure has symmetry if it is invariant under a reflection - i.e. if the effect of the reflection is to produce an identical-looking figure in the same position.
- **Vertex:** The point at which two or more lines intersect.
- **Vertical:** At right angles to the horizontal plane

Mathematical Vocabulary Progression

Geometry – Position & Direction

Year One

- **Cone:** A 3-dimensional shape consisting of a circular base, one vertex and one curved edge.
- **Direction:** The orientation of a line in space. e.g. north, south, east, west; up, down, right, left are directions.
- **Half turn:** A 180 degree rotation.
- **Left:** indicating a direction.
- **Quarter turn:** A rotation through 90° , usually anticlockwise unless stated otherwise.
- **Right:** indicating a direction.
- **Rotation:** In 2-D, a transformation of the whole plane which turns about a fixed point, the centre of rotation.
- **Turn:** A rotation about a point: a quarter turn is a rotation of 90° . A half turn is a rotation of 180° , a whole turn is a rotation of 360°

Year Two

- **Anticlockwise:** In the opposite direction from the normal direction of travel of the hands of an analogue clock.
- **Clockwise:** In the direction in which the hands of an analogue clock travel

Mathematical Vocabulary Progression

Statistics

Year One

- **Table:** An orderly arrangement of information, numbers or letters usually in rows and columns

Year Two

- **Bar chart/graph:** A format for representing statistical information.
- **Block graph:** A simple format for representing statistical information. One block = one observation.
- **Carroll diagram:** A sorting diagram in which numbers (or objects) are classified as having a certain property or not having that property.
- **Chart:** Another word for a table or graph.
- **Data:** Information of a quantitative nature consisting of counts or measurements. Initially data are nearly always counts. When they refer to measurements that are separate and can be counted, the data are discrete. When they refer to quantities such as length or capacity that are measured, the data are continuous.
- **Frequency:** The number of times an event occurs; or the number of individuals with a specific property.
- **Minimum value:** The least value.
- **Maximum value:** The largest value..
- **Pictogram:** A format for representing statistical information. Suitable pictures, symbols or icons are used to represent objects. For large numbers one symbol may represent a number of objects and a part symbol then represents a rough proportion of the number.
- **Set:** A well-defined collection of objects.
- **Sort:** To classify a set of entities into categories.
- **Tally:** Make marks to represent objects counted; usually by drawing vertical lines and crossing the fifth

Lower Key Stage Two Mathematical Vocabulary

Mathematical Vocabulary Progression

General

Year Three

- **Interpret:** Draw out the key mathematical features of a graph, or a chain of reasoning, or a mathematical model, or the solutions of an equation, etc.
- **Interval:** All possible points in the closed continuous interval between 0 and 1 on the real number line, including the end points zero and 1. Commonly used in statistics to describe the steps between two numbered points in a graph or chart axis.
- **Proof:** Using mathematical reasoning in a series of logical steps to show that if one mathematical statement is true then another that follows from it must be true.
- **Representation:** The word 'representation' is used in the curriculum to refer to a particular form in which the mathematics is presented, so for example a quadratic function could be expressed algebraically or presented as a graph; a quadratic expression could be shown as two linear factors multiplied together or the multiplication could be expanded out; a probability distribution could be presented in a table or represented as a histogram, and so on. Very often, the use of an alternative representation can shed new light on a problem. An array is a useful representation for multiplication and division which helps to see the inverse relationship between the two. The Bar Model is a useful representation of for many numerical problems

Year Four

- **Correspondence Problems:** Those in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).
- **Decimal:** Relating to the base ten. Most commonly used synonymously with decimal fractions where the number of tenths, hundredth, thousandths, etc. are represented as digits following a decimal point. The decimal point is placed at the right of the ones column. Each column after the decimal point is a decimal place.
- **Degree of accuracy:** A measure of the precision of a calculation, or the representation of a quantity. A number may be recorded as accurate to a given number of decimal places, or rounded to the nearest integer, or to so many significant figures.

Mathematical Vocabulary Progression

Number & Place Value

Year Three

- **Integer:** Any of the positive or negative whole numbers and zero. Example: ...- 2, -1, 0, +1, +2 ... The integers form an infinite set; there is no greatest or least integer.
- **Place holder:** In decimal notation, the zero numeral is used as a place holder to denote the absence of a particular power of 10.

Year Four

- **Negative number:** A number less than zero.
- **Positive number:** A number greater than zero. Where a point on a line is labelled 0 positive numbers are all those to the right of the zero and are read 'positive one, positive two, positive three' etc.
- **Roman numerals:** The Romans used the following capital letters to denote cardinal numbers: I for 1; V for 5; X for 10; L for 50; C for 100; D for 500; M for 1000. Multiples of one thousand are indicated by a bar over a letter, so for example V with a bar over it means 5000. Other numbers are constructed by forming the shortest sequence with this total, with the proviso that when a higher denomination follows a lower denomination the latter is subtracted from the former.
- **Round (verb):** In the context of a number, express to a required degree of accuracy. Example: 543 rounded to the nearest 10 is 540

Mathematical Vocabulary Progression

Addition, Subtraction, Multiplication & Division

Year Three

- **Approximation:** A number or result that is not exact. In a practical situation an approximation is sufficiently close to the actual number for it to be useful.
- **Associative law:** No matter how the parts in an addition or multiplication equation are grouped, the answer will be the same - $(6 + 3) + 2 = 11$, $6 + (3 + 2) = 11$.
- **Columnar Addition and Subtraction:** A formal method of setting out an addition or a subtraction in ordered columns with each column representing a decimal place value and ordered from right to left in increasing powers of 10
- **Commutative:** Addition and multiplication are commutative because pairs of numbers can be calculated in any order. For example, $2 + 3 = 3 + 2$ and $2 \times 3 = 3 \times 2$.
- **Compliment (in addition):** In addition, a number and its complement have a given total. Example: When considering complements in 100, 67 has the complement 33, since $67 + 33 = 100$.
- **Divisibility:** The property of being divisible by a given number. Example: A test of divisibility by 9 checks if a number can be divided by 9 with no remainder.
- **Divisor:** The number by which another is divided. Example: In the calculation $30 \div 6 = 5$, the divisor is 6. In this example, 30 is the dividend and 5 is the quotient.
- **Efficient Methods:** A means of calculation (which can be mental or written) that achieves a correct answer with as few steps as possible. In written calculations this often involves setting out calculations in a columnar layout.
- **Estimate:** A rough or approximate answer.
- **Formal written methods:** Setting out working in columnar form.
- **Near double:** See double.
- **Product:** The result of multiplying one number by another. Example: The product of 2 and 3 is 6 since $2 \times 3 = 6$
- **Quotient:** The result of a division. Example: $46 \div 3 = 15\frac{1}{3}$ and $15\frac{1}{3}$ is the quotient of 46 by 3.
- **Short division:** A compact written method of division.
- **Short multiplication:** Essentially, simple multiplication by a one-digit number, with the working set out in columns

Year Four

Mathematical Vocabulary Progression

Fractions (*including decimals and percentages*)

Year Three

- **Proper fraction:** A proper fraction has a numerator that is less than its denominator So $\frac{3}{4}$ is a proper fraction, whereas $\frac{4}{3}$ is an improper fraction (i.e. not proper).
- **Vulgar fraction:** A fraction in which the numerator and denominator are both integers. Also known as common fraction or simple fraction

Year Four

- **Cancel (a fraction):** One way to simplify a fraction down to its lowest terms. The numerator and denominator are divided by the same number e.g. $\frac{4}{8} = \frac{2}{4}$. Also to 'reduce' a fraction.
- **Common fraction:** A fraction where the numerator and denominator are both integers. Also known as simple or vulgar fraction.
- **Decimal fraction:** Tenths, hundredths, thousandths etc represented by digits following a decimal point.
- **Improper fraction:** An improper fraction has a numerator that is greater than its denominator.
- **Simplify:** reduce a fraction to its simplest form

Mathematical Vocabulary Progression

Measurement

Year Three

- **Convert:** Changing from one quantity or measurement to another.
- **Kilogram:** Symbol: kg. The base unit of mass in the SI (Système International d'Unités). $1\text{kg.} = 1000\text{g}$
- **Kilometre:** Symbol: km. A unit of length in the SI (Système International d'Unités). The base unit of length in the system is the metre. $1\text{km.} = 1000\text{m}$
- **Metre:** Symbol: m. The base unit of length
- **Perimeter:** The length of the boundary of a closed figure

Year Four

- **Area:** A measure of the size of any plane surface. Area is usually measured in square units e.g. square centimetres (cm^2), square metres (m^2)
- **Digital Clock:** A clock that displays the time as hours and minutes passed, usually since midnight. Example: four thirty in the afternoon is displayed as 16:30.
- **Rectilinear:** Bounded by straight lines. A closed rectilinear shape is also a polygon. A rectilinear shape can be divided into rectangles and triangles for the purpose of calculating its area.
- **Square centimetre:** Symbol: cm^2 . A unit of area, a square measuring 1 cm by 1 cm. $10000\text{ cm}^2 = 1\text{ m}^2$.
- **Square metre:** Symbol: m^2 . A unit of area, a square measuring 1m by 1 m

Mathematical Vocabulary Progression

Geometry – Properties of Shapes

Year Three

- **Acute angle:** An angle between 0° and 90° .
- **Angle:** An angle is a measure of rotation and is often shown as the amount of rotation required to turn one line segment onto another where the two-line segments meet at a point.
- **Centre:** The middle point for example of a line or a circle.
- **Composite shape:** A shape formed by combining two or more shapes.
- **Cross-section:** In geometry, a section in which the plane that cuts a figure is at right angles to an axis of the figure. Example: In a cube, a square revealed when a plane cuts at right angles to a face.
- **Decagon:** A polygon with ten sides and ten angles.
- **Horizontal:** Parallel to the horizon.
- **Heptagon:** A polygon with seven sides and seven edges.
- **Hexagon:** A polygon with six sides and six edges.
- **Irregular:** In geometry, irregular is a term used to describe shapes that are not regular.
- **Obtuse angle:** An angle greater than 90° but less than 180° .
- **Octahedron:** A polyhedron with eight faces. A regular octahedron has faces that are equilateral triangles.
- **Parallel:** denotes two lines that are always equidistant
- **Perpendicular:** A pair of line segments (or surfaces) can be described as perpendicular if they intersect at (or form) a right angle
- **Polyhedron:** A closed solid figure bounded by surfaces (faces) that are polygonal. Its faces meet in line segments called its edges. Its edges meet at points called vertices.
- **Right:** Used as an adjective, right-angled or erect.
- **Right angle:** One quarter of a complete turn. An angle of 90 degrees. An acute angle is less than one right angle. An obtuse angle is greater than one right angle but less than two. A reflex angle is greater than two right angle

Year Four

- **Equilateral triangle:** A triangle where all sides are of equal length and angles are the same.
- **Isosceles:** Isosceles triangles have two equal sides
- **Kite:** A quadrilateral with two pairs of equal, adjacent sides whose diagonals consequently intersect at right angles.
- **Parallelogram:** A quadrilateral whose opposite sides are parallel and consequently equal in length.
- **Reflection:** In 2-D, a transformation of the whole plane involving a mirror line or axis of symmetry in the plane.
- **Regular:** Describing a polygon, having all sides equal and all internal angles equal.
- **Rhombus:** A parallelogram with all sides equal.
- **Scalene:** A scalene triangle has three unequal sides and three unequal angles
- **Tetrahedron:** A solid with four triangular faces. A regular tetrahedron has faces that are equilateral triangles
- **Trapezium:** A quadrilateral with exactly one pair of sides parallel

Mathematical Vocabulary Progression

Properties of Shape – Position and Direction

Year Three

Year Four

- **Axis:** A fixed, reference line along which or from which distances or angles are taken.
- **Coordinates System:** A system used to define the position of a point in two- or three-dimensional space. Two axes at right angles to each other are used to define the position of a point in a plane. The usual conventions are to label the horizontal axis as the x-axis and the vertical axis as the y-axis with the origin at the intersection of the axes. The ordered pair of numbers (x, y) that defines the position of a point is the coordinate pair. The origin is the point (0,0); positive values of x are to the right of the origin and negative values to the left, positive values of y are above the origin and negative values below the origin. Each of the numbers is a coordinate.
- **Coordinate:** In geometry, a coordinate uniquely determines the position of a point in space on a grid, using an x coordinate and a y-coordinate
- **Grid:** A lattice created with two sets of parallel lines
- **Origin:** A fixed point from which measurements are taken. In coordinates, denoted as (0,0)
- **Plot:** The process of marking points. Points are usually defined by coordinates and plotted with reference to a given coordinate system.
- **Translation:** A transformation in which every point of a body moves the same distance in the same direction. A transformation specified by a distance and direction

Mathematical Vocabulary Progression

Statistics

Year Three

- **Column:** A vertical arrangement for example, in a table the cells arranged vertically.
- **Graph:** A diagram showing a relationship between variables

Year Four

- **Continuous Data:** Data arising from measurements taken on a continuous variable (examples: lengths of caterpillars; weight of crisp packets) .
- **Line Graph:** A graph in which adjacent points are joined by straight-line segment

Upper Key Stage Two Mathematical Vocabulary

Mathematical Vocabulary Progression

Number & Place Value

Year Five

- **Power (of 10):** 100 (i.e. 10^2 or 10×10) is the second power of 10, 1000 (i.e. 10^3 or $10 \times 10 \times 10$) is the third power of 10 etc
- **Prime factors:** The factors of a number that are prime.
Example: 2 and 3 are the prime factors of 12 ($12 = 2 \times 2 \times 3$).
- **Prime factor decomposition:** The process of expressing a number as the product of factors that are prime numbers.
Example: $24 = 2 \times 2 \times 2 \times 3$ or $2^3 \times 3$. Every positive integer has a unique set of prime factors

Year Six

Mathematical Vocabulary Progression

Addition, Subtraction, Multiplication & Division

Year Five

- **Common Factors:** A number which is a factor of two or more other numbers, for example 3 is a common factor of the numbers 9 and 30.
- **Common multiple:** An integer which is a multiple of a given set of integers, e.g. 24 is a common multiple of 2, 3, 4, 6, 8 and 12.
- **Cube number:** A number that can be expressed as the product of three equal integers. Example: $27 = 3 \times 3 \times 3$. Consequently, 27 is a cube number; it is the cube of 3 or 3 cubed. This is written compactly as $27 = 3^3$, using index, or power, notation.
- **Factor:** When a number can be expressed as the product of two numbers, these are factors of the first. examples: 1, 2, 3, 4, 6 and 12 are all factors of 12 because $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$
- **Factorise:** To express a number as the product of its factors. Examples: Factorising 12: $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$.
- **Remainder:** In the context of division requiring a whole number answer (quotient), the amount remaining after the operation. Example: 29 divided by 7 = 4 remainder 1.
- **Square (multiplication):** the square of a number is the product of the number and itself.
- **Square number:** A number that can be expressed as the product of two equal numbers. Example $36 = 6 \times 6$ and so 36 is a square number or "6 squared". A square number can be represented by dots in a square array

Year Six

- **Level of accuracy:** Often in reference to the number of significant figures with which a numerical quantity is recorded and made more precise by stating the range of possible error. The degree of precision in the measurement of a quantity.
- **Long Division:** A columnar method for division by more than a single digit.
- **Order of operation:** This refers to the order in which different mathematical operations are applied in a calculation

Mathematical Vocabulary Progression

Fractions (including decimals and percentages)

Year Five

- **Mixed Fraction:** A whole number and a fractional part expressed as a common fraction. Example: $1\frac{1}{3}$ is a mixed fraction. Also known as a mixed number.
- **Mixed Number:** A whole number and a fractional part expressed as a common fraction. Example: $2\frac{1}{4}$ is a mixed number. Also known as a mixed fraction.
- **Percentage:** A fraction expressed as the number of parts per hundred and recorded using the notation %

Year Six

- **Recurring decimal:** A decimal fraction with an infinitely repeating digit or group of digits. Example: The fraction $\frac{1}{3}$ is the decimal 0.33333 ..., referred to as nought point three recurring and may be written as 0.3 (with a dot over the three). Where a block of numbers is repeated indefinitely, a dot is written over the first and last digit in the block e.g. $1/7 = 0.\dot{1}42857\dot{}$

Mathematical Vocabulary Progression

Ratio and Proportion

Year Five

Year Six

- **Proportion:** A part to whole comparison. Example: Where £20 is shared between two people in the ratio 3 : 5, the first receives £7.50 which is $\frac{3}{8}$ of the whole £20. This is his proportion of the whole.
- **Ratio:** A part-to-part comparison. The ratio of a to b is usually written a : b. Example: In a recipe for pastry fat and flour are mixed in the ratio 1 : 2 which means that the fat used has half the mass of the flour, that is amount of fat/amount of flour = $\frac{1}{2}$. Thus ratios are equivalent to particular fractional parts.
- **Scale:** To enlarge or reduce a number, quantity or measurement by a given amount (called a scale factor). e.g. to have 3 times the number of people in a room than before; to find a quarter of a length of ribbon; to find 75% of a sum of money.
- **Scale factor:** For two similar geometric figures, the ratio of corresponding edge lengths

Mathematical Vocabulary Progression

Algebra

Year Five

- **Algebra:** The part of mathematics that deals with generalised arithmetic. Letters are used to denote variables and unknown numbers and to state general properties. Example: $a(x + y) = ax + ay$ exemplifies a relationship that is true for any numbers a , x and y .
- **Formula:** An equation linking sets of physical variables. e.g. $A = \pi r^2$ is the formula for the area of a circle

Year Six

- **Equivalent expression:** A numerical or algebraic expression which is the same as the original expression, but is in a different form which might be more useful as a starting point to solve a particular problem. Example: $6 + 10x$ is equivalent to $2(3 + 5x)$.

Mathematical Vocabulary Progression

Measurement

Year Five

- **Imperial unit:** A unit of measurement historically used in the United Kingdom and other English-speaking countries. Units include inch, foot, yard, mile, acre, ounce, pound, stone, hundredweight, ton, pint, quart and gallon. Now largely replaced by metric units.
- **Inch:** Symbol: in. An imperial unit of length. 12 inches = 1 foot. 36 inches = 1 yard. Unit of area is square inch, in². Unit of volume is cubic inch, in³. 1 inch is approximately 2.54 cm
- **Metric unit:** Unit of measurement in the metric system. Metric units include metre, centimetre, millimetre, kilometre, gram, kilogram, litre and millilitre.
- **Millilitre:** Symbol: ml. One thousandth of a litre.
- **Pint:** An imperial measure of volume applied to liquids or capacity. In the imperial system, 8 pints = 4 quarts = 1 gallon. 1 pint is just over 0.5 litres
- **Pound (mass):** Symbol: lb. An imperial unit of mass. In the imperial system, 14 lb = 1 stone. 1 lb is approximately 455 grams. 1 kilogram is approximately 2.2 lb

Year Six

- **Cubic centimetre:** Symbol: cm³. A unit of volume. The three-dimensional space equivalent to a cube with edge length 1cm.
- **Cubic metre:** Symbol: m³. A unit of volume. A three-dimensional space equivalent to a cube of edge length 1m.
- **Foot:** Symbol: ft. An imperial measure of length. 1 foot = 12 inches. 3 feet = 1 yard. 1 foot is approximately 30 cm.
- **Gallon:** Symbol: gal. An imperial measure of volume or capacity, equal to the volume occupied by ten pounds of distilled water. In the imperial system, 1 gallon = 4 quarts = 8 pints. One gallon is just over 4.5 litres.
- **Mile:** An imperial measure of length. 1 mile = 1760 yards. 5 miles is approximately 8 kilometres.
- **Ounce:** Symbol: oz. An imperial unit of mass. In the imperial system, 16 ounces = 1 pound. 1 ounce is just over 28 grams.
- **Rate:** A measure of how quickly one quantity changes in comparison to another quantity. For example, speed is a measure of how distance travelled changes with time; the average speed of a moving object is the total distance travelled/ time taken to travel that distance.
- **Yard:** Symbol: yd. An imperial measure of length. In relation to other imperial units of length, 1 yard = 3 feet = 36 inches. 1760yd. = 1 mile One yard is approximately 0.9 metres

Mathematical Vocabulary Progression

Geometry – Property of Shapes

Year Five

- **Angle at a point:** The complete angle all the way around a point is 360° .
- **Angle at a point on a straight line:** The sum of the angles at a point on a line is 180° .
- **Degree:** The most common unit of measurement for angle.
- **Diagonal:** A line segment joining any two non-adjacent vertices of a polygon.
- **Dodecahedron:** A twelve-sided polygon.
- **Exterior angle:** angle on the outside of a shape.
- **Interior angle:** angle on the inside of a shape.
- **Protractor:** An instrument for measuring angles.
- **Reflex angle:** greater than 180 degrees.
- **Set square:** A drawing instrument for constructing parallel lines, perpendicular lines and certain angles.
- **Vertically opposite angles:** The pair of equal angles between two intersecting straight lines. There are two such pairs of vertically opposite angles

Year Six

- **Arc:** A portion of the circumference of a circle
- **Circumference:** The distance around a circle (its perimeter)
- **Compasses:** A tool for creating curved lines, arcs and circles.
- **Diameter:** Any of the chords of a circle or sphere that pass through the centre.
- **Net:** A plane figure composed of polygons which by folding and joining can form a polyhedron.
- **Radius:** In relation to a circle, the distance from the centre to any point on the circle.

Mathematical Vocabulary Progression

Geometry – Position & Direction

Year Five

- **Quadrant:** One of the four regions into which a plane is divided by the x and y axes in the Cartesian coordinate system

Year Six

- **Intersect:** the point at which two lines meet

Mathematical Vocabulary Progression

Statistics

Year Five

Year Six

- **Average:** Loosely an ordinary or typical value, however, a more precise mathematical definition is a measure of central tendency which represents and or summarises in some way a set of data.
- **Mean:** Often used synonymously with average. The mean (sometimes referred to as the arithmetic mean) of a set of discrete data is the sum of quantities divided by the number of quantities.
- **Pie chart:** Also known as pie graph. A form of presentation of statistical information. Within a circle, sectors like 'slices of a pie' represent the quantities involved. The frequency or amount of each quantity is proportional to the angle at the centre of the circle.